

OPEN ACCESS

Klein bottle logophysics: a unified principle for non-linear systems, cosmology, geophysics, biology, biomechanics and perception

To cite this article: Diego Lucio Rapoport 2013 *J. Phys.: Conf. Ser.* **437** 012024

View the [article online](#) for updates and enhancements.

Related content

- [Isometric immersions and embeddings of a flat Möbius strip in Euclidean spaces](#)
Idzhad Kh Sabitov
- [Leonardo da Vinci, physicist](#)
- [VSD imaging of ICMS-evoked neural activity in cat visual cortex](#)
Jun Suzurikawa, Toshiki Tani, Masayuki Nakao et al.

Recent citations

- [The mathematics of Soft logic](#)
Moshe Klein and Oded Maimon

Klein bottle logophysics: a unified principle for non-linear systems, cosmology, geophysics, biology, biomechanics and perception

Diego Lucio Rapoport

Department of Sciences and Technology, Universidad Nacional de Quilmes, Bernal,
Buenos Aires, ARGENTINA

E-mail: diego.rapoport@gmail.com

Abstract. We present a unified principle for science that surmounts dualism, in terms of torsion fields and the non-orientable surfaces, notably the Klein Bottle and its logic, the Möbius strip and the projective plane. We apply it to the complex numbers and cosmology, to non-linear systems integrating the issue of hyperbolic divergences with the change of orientability, to the biomechanics of vision and the mammal heart, to the morphogenesis of crustal shapes on Earth in connection to the wavefronts of gravitation, elasticity and electromagnetism, to pattern recognition of artificial images and visual recognition, to neurology and the topographic maps of the sensorium, to perception, in particular of music. We develop it in terms of the fundamental 2:1 resonance inherent to the Möbius strip and the Klein Bottle, the minimal surfaces representation of the wavefronts, and the non-dual Klein Bottle logic inherent to pattern recognition, to the harmonic functions and vector fields that lay at the basis of geophysics and physics at large. We discuss the relation between the topographic maps of the sensorium, and the issue of turning inside-out of the visual world as a general principle for cognition, topological chemistry, cell biology and biological morphogenesis in particular in embryology

1. Introduction

The objective of this article is to present a conceptual elaboration of fundamental issues of physics, namely the relation between divergences in hyperbolic geometry and non-linear science, to torsion fields and non-orientable topologies (the Möbius strip, the Klein Bottle and the projective plane) in several fields: cosmology, non-linear evolution equations and geophysics, to further apply them to biomechanics, perception, biology and chemistry. It is the nature of this paradigm and its current presentation in this article, that it can be read (presented) starting anywhere in its extension, alike to James Joyce *Finnegans Wake*, to return to this departing point finding the foundational elements once and again in the percourse of the presentation. Author Samuel Beckett has traced Joyce's recursive wake to the cyclical structure of Giambattista Vico's seminal text *Scienza Nuova (New Science)*, Newton's contemporary, pioneer investigator of complexity and systemics, critical of Cartesian dualism, whose surmountal is very much the issue of this article.

Physics has developed since Newton associated to a dualistic framework; the Third Law already is framed in terms of action and reaction; particles and antiparticles and CPT symmetry, are basic in quantum field theory. Einstein developed General Relativity with the conception of an absolute space,



independent of self-reference and of the structure and processes of subjective experience, to which time, originally absolute in Newton's physics, became a variable related to space through the light-cone and the Lorentz invariance, while relativistic quantum mechanics indicates that it is the double covering of the Lorentz group the symmetry in question. Thus, two dimensions –though complex- are at stake; yet, this is also the case of the paradigm to be presented here, in which space does no longer hold the autonomous of experience character ascribed to it in physics. Time will turn out to be associated to depth, a phenomenological –and still, physiological- primal variable unseparable from non-dual logic by which, through self-penetration, the world manifests itself [14,134,135].

Biology has taken a similar dualistic stance, on facing the problem of what is life, first raised by Schrödinger [1]. The organization of the cell, the development of the embryo and morphogenesis, when elaborated from a Newtonian perspective of physics which is very much resisted by biologists of the molecular biology persuasion, has also taken dualism as its basic cornerstone [2,3,65]. Biophysicists dealing with the buildup of organisms from the spherical mother cell, will start by claiming that they are spherical [2,3,4], or still, to others those organisms having bilateral symmetry are claimed to be toroidal [5,6]. Both are topologies which in a static conception the division of the world is already established in terms of the outside and inside, as is the case of the embryonic sphere. This division is done in disregard that algae *Volvox Carteri*, which developmental biologists have come to consider the case study for morphogenesis, turns inside-out, a fact [7] that mathematicians have already established in the so-called eversion of the 2-dimensional sphere [8]. Yet, in a dynamic conception, it is the case that through smooth transformations, the sphere can be turned inside-out (i.e. everted). This is the Smale Paradox of topology; it is not restricted to a sphere, a torus can also be everted. Yet, it is important to remark, that although the sphere and the torus, are orientable surfaces, the eversion requires that the turning inside-out is carried through *half-way* surfaces which are non-orientable; otherwise, it is conceptually impossible, though a detailed proof is lacking, to evert a surface. This is the case of eversion of the sphere proposed by mathematician Morin, which has Möbius strips in the half-way transformation of Inside and Outside, and also is the case of a half-way eversion through the Boy surface, which is the embedding in Euclidean 3D space of the non-orientable projective plane. Yet, a remarkable precursor to eversions turning inside-out a sphere or a torus, is the Klein Bottle surface, a two-dimensional self-contained manifold for which there is no global inside nor global outside but a dynamic holonomic structure which processually turns inside-out and outside-inside. For this surface there is no half-way surface to produce the transformation; it is its very nature to do so, in its construction and in its form; neither it is a matter of statics nor of dynamics to produce this transformation, but rather an issue of holonomy, in a sense which had for precursor the theory of quantum mechanics due to David Bohm [94], yet which integrates his implicate and manifest realms [14].

In a series of articles, it has been demonstrated that the Klein Bottle has a crucial role in biology (embryological development and differentiation [9,10], cellular biology [11], the genetic code and its relation with the Dirac algebra of Quantum Mechanics [12], the problem of what is life [9], biochemical recognition and a quantum theory of chemistry in terms of Bohm's theory of Quantum Mechanics [13], perception and anatomy-physiology [9,14,15,27-32], cybernetics and systemics [9,14], cognition and perception [9,15]. This appeared to be the case without positing a claim for totality as a petition of principle, but the derivation of seemingly unconnected scientific disciplines from a common unifying principle: The Klein Bottle as the embodiment of self-referentiality, rather than the petition of principle of dualism. Instead of an objective world as the usual take of theoretical physics would have it, the issue at stake is that of the experienced world and its joint constitution with the subject [140], enacted by active participation unrestricted to observation, thus superseding the conceptual framework of the Copenhagen interpretation of quantum physics.

Thus, whether this principle embraces *all* of reality or fails to do so, is by its very self-referential nature tied to this enactment rather than being an a priori manifesto: this is a physical (unrestricted to the reductive sense of the science of physics), sensorial, logical and cognitive bootstrapping which does not allow for an absolute framework other than self-reference. It is communicated through language,

which whatever its quality may be, it rests on signs and utterances either generic (including formal languages as is the case of mathematics and its signs) or particular as applied to measurements which are designed by language themselves, thus having the same inherent self-referential being of this principle.

Allness is unseparable of self-referentiality, as Bertrand Russell would discover in the paradox later named after him, which lead him to pursue the provision of a dualistic basis for mathematics and metamathematics in his *Principia Mathematica* written with Whitehead, attempting to banish paradox of naive set theory and to banish paradox altogether. In this effort he was joined (Frege also played a central role) by mathematician and philosopher, founder of phenomenology, Edmund Husserl; (see [132,131], in particular his discussion on wholes and parts). Set theory is defined in dualistic terms, to belong or not to belong, and thus its blending with self-reference as in Russell's Paradox (namely, let R be the set of all sets that are not members of themselves; is R a member of R ?) would manifest the paradoxical character of this blending. Set theory is non-processive but structurally static, and thus unappropriate to a conception of science in which self-reference is its foundational principle, and unsuitable for phenomenology as a philosophy of the lived world. Remarkably, this appears to have been altogether unconsidered by Husserl [131,132] and fully acknowledged by Spencer-Brown in his *Laws of Form* [18], to which we shall return below; see also the dualistic theory of autopoiesis in system theory, cognition science and biology [125] following Spencer-Brown, and the theory of ontopoiesis and the problem of what is life [9] elaborating the current paradigm.

To conclude, the universality of the paradigm hereby to be presented is unseparable from the universality of self-reference; it is independent of any statement based on quantifiers and independent as well of the notion of an elusive completeness which is beyond verification, if not through the lived world [9].

The paradigm hereby presented operates providing a non-dual multistate logic - and thus paradoxical states are inherent to this logic (as precursors of quantum superposition states): the Klein Bottle logic [9]. It has a Matrix Logic representation [14-16] which has quantum, fuzzy and Boolean logics for particular cases, and allows to demonstrate that dualism is nothing but a projection of the superposed topological states (produced by non-orientability of surfaces) to the Boolean states, and still to recover from them the superposed states; in other words, dualism is not a primeval independent logic but a projection of the Klein Bottle logic. This may provide a unifying principle for science and further integrate it with the analytical philosophical *tradition* (of which Russell and Frege were paramount exponents) that claimed to the possibility of establishing a relation between philosophy and science based on the latter, yet which its formalization was proved to be obsolete and misguided, through its *self-referential* reformulation in the Calculus of forms due to Spencer-Brown [18]. Thus, paradox and self-referentiality that the analytical traditional wished to banish (and Gödel Theorem remarked that it was impossible to do so for the natural numbers) was found to be the very roots of a logic with imaginary values, hitherto absent in logic, crucial to this paradigm.

The fundamental idea is that a sole sign of distinction, provided with two simple rules of generation (one of them being formal nilpotence, say of the exterior differential of calculus of functions of many variables real or complex) in which the sign is both operator and operand, allows not only to generate a formal system (free of logical quantifiers), but also time waves that stand for the imaginary logical values realizing the reentrance of the system on itself. This sign was further associated to the torsion geometry of spacetime [14,15], which is already crucial to the current paradigm of physics in terms of symmetries, since in the theory of Lie groups their structural coefficients are the Christoffel coefficients of a torsion geometry. In the Matrix Logic formulation of the Klein Bottle logic, this torsion appears through the lack of commutativity of the True and False operators embodied in a cognition operator through which cognitive statements are translated into quantum statements for quantum states, and in the photon field, in particular associated to singularities that sustain quantum jumps ([19,20,15] and the decomposition of the cognitive vacuum operator given by the null operator in Matrix Logic into twistors, thus unifying further the cognitive realm with rotational structures, which are further embodied as superposition states of the logic. In other words, the quantum and

cognitive realms become integrated through vortical structures, whose topology is that of the Möbius strip and the Klein Bottle. So, torsion fields unseparably of non-orientable surfaces play a crucial role in this nascent paradigm of unification, providing the self-referential geometry of spacetime, cognition and logic, and still to the forms of life [21,22] and crucial to biology at large [9,10,11,12].

In the case of perception and the physiology of organisms, the division of Outside and Inside is also the case. In particular, the division of the world into the Exterior and Interior of a system, is very much ingrained in the sciences. Philosophers like Kant have tried to surmount this division by proposing noumenic and phenomenic worlds, which would somehow account for what he conceived to be a division between what there is and what can be known, in this case, of the world at large; in philosophical parlance, a division between ontology (what there is) and epistemics (how do we come to know about the world). Thus, for Kant, perception is the representation by our senses of the world out there, which is impossible to be fully cognized as it is, so we are constrained to represent a fleeting world, as from inside our senses and our cultural background as well. In fact, it appears to be untenable to prove that what we perceive is an outer world or an interior one, for that matter; cognition scientists have come to propose that instead of seeing an outer world, what we actually see is our brains [23], and thus it becomes impossible to establish a clear cut between the individual brain and that of the others. Indeed, unless we follow infinite regress, there is no absolute cognitive framework other than self-reference. This latter notion is widely unacknowledged –if not resisted- in science and in particular, in relation to consciousness as an epiphenomenon, an emergence that is postulated to originate in non-computability originating in quantum processes on the brain, which is formulated in terms of self-reference, yet denying it an ontological or epistemological principle [111].

We have argued that the photon is the ultimate physical process of self-referential nature and the eikonal equations of propagation of wavefronts of electromagnetic signals, to be the embodiment of self-reference [14,15,20]. This is rather general and even has a wide material manifestation. Indeed, Fock argued that the theory of relativity was conceptually wrong and that the ultimate reference frame to construct a theory of gravitation is provided by the solutions to the eikonal equation of the wavefront propagation of electromagnetic signals, i.e. fields that are discontinuous [111]. These wavefront propagation of electromagnetic fields are null ahead of the wavefront and are discontinuous in the wavefront, which depends on the higher order of the derivatives, two, and thus will present first order discontinuities; a spinor topological model of the photon was built as an electromagnetic signal –in the sense of Fock- by Kiehn [136]. They define, according to Fock, a universal reference frame, in terms of which physical reality comes to be [111]. Yet, due to the limitations of speed of light, for *any* field that has a limiting velocity of propagation, *indistinctly of the propagation being in the vacuum or in a matter field*, the equation for the wavefront propagation (the characteristics of the hyperbolic equations) is the eikonal equation, which rather than defining an objective geometry, they are related to torsion, which is essentially self-referential as the photon is –the photon is not seen but the seeing process [14,15,20]. So, *whether it is a propagation of elasticity, electromagnetic or gravitational fields, either on the vacuum or in matter*, say the surface of Earth, or the interior of a differentiating cell [11,2], they all have the same form of signalling their existence and manifestation in terms of the eikonal equations for light rays and for a physical manifestation of the singularities, the appearance of quantum jumps [19]. In fact, these signals are invariant by the Lorentz group transformations, so they are universal in their self-referentiality. This Lorentz group invariance, was argued by Fock to be the basis for a theory of gravitation; this group can be derived from the algebra of forms in Spencer-Brown's calculus [133]. These fields will all develop discontinuities on their propagating wavefronts which we shall see play a fundamental morphogenetic role, in particular on Earth's crust, as we proved it was the case of biological cells under differentiation. In this conception, this morphogenesis which manifests the diversity of reality as forms, has a self-referential origin, which is unconceivable for dualism, and still originates the physical world. In other words, rather than the emergence of consciousness and morphology, it is self-referentiality their source and that of the material world [9,14,15].

The physics and mathematics of perception has already a tradition, established in the 19th century by Fechner and von Helmholtz [24], that has been developed until present times, in particular in music [25], in vision [25,26,31,32] and pattern recognition [26,27], and in more general terms, in the search by neurologists of topographical maps of the sensorium, i.e. the existence of mathematical maps of the exterior sensorial input on our brains [27,28,29,30,31], mediated by the body [9,14,15,29]. In terms of the development of an early embryonic cell, the brain is by no means an interior to it, but a turning inwards of the *ectoderm* (exterior tissue), so this qualification of topographical maps being representations of an outer world in our brains as if the locus of our interiority, appears to be nothing but more of a the imposition of dichotomic thinking, which in this case, it turns out to be unwarranted, if not overtly wrong. It becomes patent that this conception is based on statics, and not in developmental considerations, the dynamics of forms and the problem of morphogenesis. They are the subject of non-linear science [33], and in particular, of the self-organization of systems, in particular, chemical, as dishpans convection experiments clearly shows, not to mention the generation of patterns from sound as empirically confirmed by Jenny's Cymatics [35] (with Galileo and Chaldni for predecessors), discussed in Rayleigh's treaty on sound [33].

In terms of number theory, the extension of the natural numbers to the integers, assume a duality and a non-compactification of them, which in placing the number system of measurements in terms of the real numbers (a practical impossibility, since cut-offs are inexorable) which Quantum Mechanics establishes to be the realm of what is being measured due to the use of self-adjoint operators whose spectra are real numbers. This is the case despite the fact that the complex numbers are essential to physics as the phase functions, yet appearing already in the classical harmonic operators, that linear combinations of the complex solutions of the characteristic polynomials of the differential equations allows to obliterate the square root of minus 1 in the solutions while keeping the oscillatory motions.

Yet, most surprisingly ignored at large, is the fact that the linear superposition of waves, not only modulates the amplitude of the resultant wave, but also produces a modulation of the phase, that is manifested as a time shift associated to the interference patterns. This allows to make away with the complex numbers for the purpose of writing the wave created by the superposition, which can be thus written as the product of sinusoidal functions, yet introducing anticipatory behaviour by this phase modulation [36]. We have discussed already the relation between the Klein Bottle and anticipatory phenomenae [9,10,11,12,14,15,130] yet in this article, the issue will be presented in another way which has been very much overlooked, apparently even by astronomer Möbius himself, a fundamental implicit harmonic of the holonomy of these non-orientable topologies. Yet, this modulation of phase in the case of linear superposition of waves, will turn out to be of crucial importance with regards to visual perception and pattern recognition, to the crustal morphogenesis of Earth, to anatomy-physiology, again related to the Klein Bottle logic.

2. The projective plane, the Möbius strip and the gnomonic projection

Complex analysis does away with the dual division between the positive and negative numbers, by representing the extended complex plane in the compact Riemann sphere, and thus the complex plane and the real numbers, by construction, become compactified, and in the associated stereographic projection, infinity is no allegory but the North Pole, and thus infinity and minus infinity become a single point. This point is both positive and negative, and neither one, and thus in terms of the Matrix Logic form of the Klein Bottle logic, it corresponds to the non-orientability Möbius strip states corresponding to topological and logical superposition [14,15,16]. Hence, by recourse of transforming the flat complex plane to the constantly curved sphere, the duality between negative and positive numbers becomes surmounted. Thus, the imposition of duality that is basic to contemporary approaches to quantum mechanics, is left wanting, since the number system which is basic to it, is not dual, when represented in the Riemann sphere. Yet, we can use this sphere, by further identifying the antipodal points through the sphere's origin, thus yielding a compact representation of the projective space. In this case, infinity is again no allegory for divergences, but the Equator. In fact, now we have

embedded in the sphere, with antipodal points identified, the Möbius band, so non-orientability is the case, *locally*.

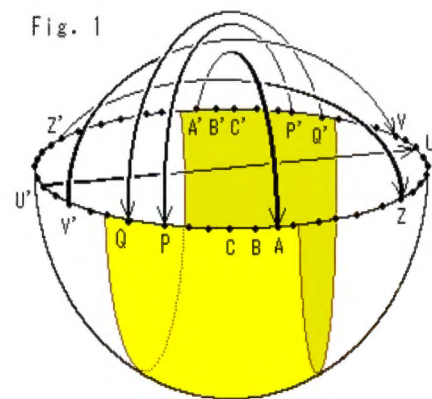


Fig. 1

Figure 1. Two points at opposite ends of a sphere are called **antipodal points**. On the edge of a hemisphere there are countless anti-podal points. All antipodal points on the edge of the hemisphere made by central projection from the ground must be identified. That is, the ground that is spreaded infinitely has something like antipodal points. Such plane is projective plane. Though a visual model of the entire projective plane can not be made, we can get a part of it by cutting the hemisphere like the yellow belt. It is a well-known **Möbius strip**

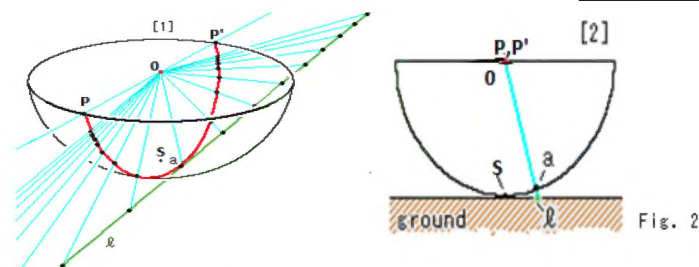


Fig. 2

Look at Figs. 2 (courtesy of T. Ito) [1]: The red spot is the center of hemisphere O . The black point S is its bottom (South Pole) where the ground plane touches. The green line l is a straight line drawn on the ground. The edge of the hemisphere O is the line at infinity. The sky blue lines are projection rays from the green straight line l to the surface of the semisphere. The red line PP' is the projected line that is a great circle. Point a is the middle point on line PP' . This red line PP' should be regarded as a straight line that stretches infinitely. We are going to see what happens if we put it onto Möbius strip. [2]: It is the cross section passing three points O , a and S for clarifying their positional relation. They are always in the positional relation as shown. At this moment, point a is apart from point S and it is not on green line l . Naturally, in a case, point a is on point S and so is on green line l .

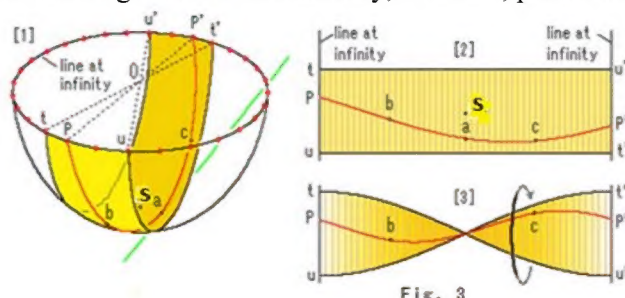


Fig. 3

Look at Fig. 3. [1]: The red spots on the edge (line at infinity) of hemisphere O show countless antipodal points. The dotted lines indicate that the end points of red line PP' (= straight line) are antipodal and points t, t' and points u, u' are also antipodal. We cut hemisphere O as shown. It is convenient for construction to cut hemisphere O in perpendicular to the equatorial plane. But you may freely cut hemisphere O as long as the red line PP' is in between lines tt' and uu' . Marks a, b, c and G are to trace points on hemisphere O to Möbius strip. [2]: It is a rectangular strip taken from hemisphere O . We treated the cut-out as if it is rubber made, and smoothed it flat. Since both end lines, right and left are at infinity, we may regard the length

of rectangular strip is infinity. But the lines at infinity is that we pulled in our hands [3]: The right end of [2] is twisted by 180° while the left is fixed.

We continue to Figure 4. [4]: It is a view from top of [3]. [5]: We jointed the end lines of [3] and got Möbius strip. The line at infinity is the joint line of Möbius strip, and point **a** is on the opposite side. [6]: We newly cut Möbius strip at point **a** and flat it. This time, the red line is smoothly connected. Thus we caught how a straight line that extends to infinity makes a loop at infinity. And the red “straight line” on Möbius strip that is a curved surface keeps its character as a straight line. No one knows whether the straight line acts like this at infinity or not. However, we feel nothing illogical. Naturally, It is based on an idea that there actually exists what is called infinity and a straight line can be stretched to that infinity.

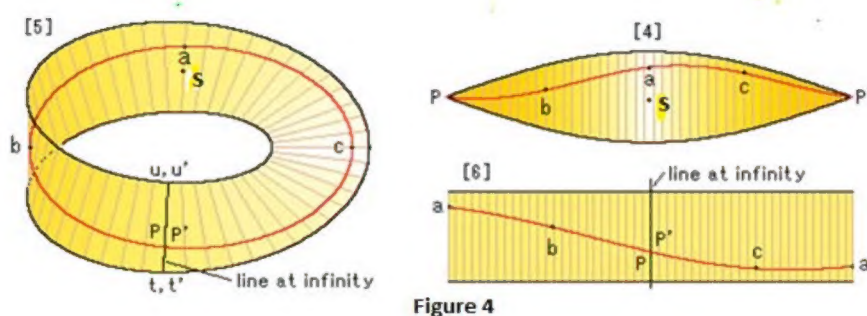
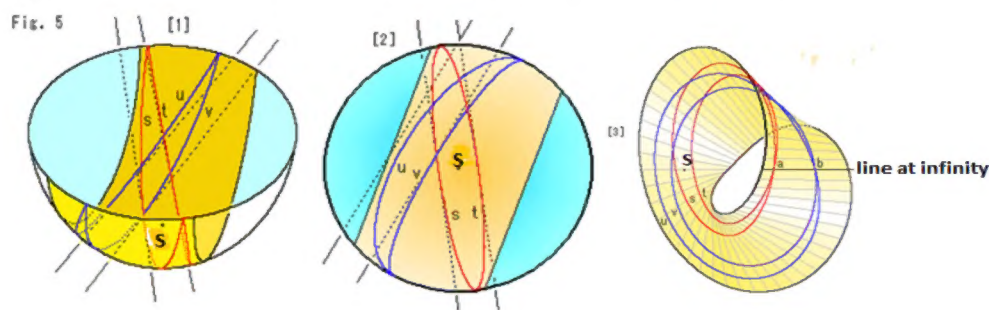


Figure 4

So far we have seen how the non-orientable Möbius strip is locally embedded in the projective plane and that infinity has a well defined meaning. Yet, the projective geometry is associated to hyperbolic geometry. Indeed, consider Figure 5 (courtesy of T. Ito):



Two pairs of parallel lines now.[1]: We have to cut the hemisphere so widely that two pairs of parallel lines are contained. On the ground the red parallel lines are just under the hemisphere but the blue parallel lines are off to the side.[2]: It is the bird's-eye view of [1].[3]: It is Möbius strip made from [2]. It is drawn to show both of the line at infinity and the area where the two pairs cross each other are facing to us at a time. On the line at infinity, the red parallel lines meet at point **a** and the blue parallel lines meet at point **b**. The distance between points **a** and **b** is infinity though it is visible. Therefore we have to regard not only the length but also the width of Möbius strip as infinity. There is no ordinary distance on the line at infinity.

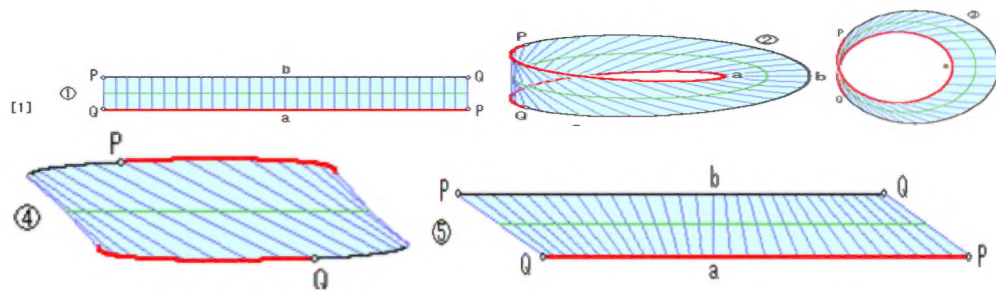


Figure 6: [1]: It is in case that the length of edges of the rectangular strip is identical to that of Möbius strip. ① We mark the rectangular strip into n equal parts, and draw the green center line. Points P , Q are to be connected for Möbius strip so that they are marked with a for attracting attention. One side of the edge of rectangular strip is colored in red to see which is which part on Möbius strip. ② The marked lines are broken at the center line. They shall be smoothly curved if we draw them accurately. But we omitted on purpose for attract attention. Compared with the rectangular strip. ①, the figure is enlarged. ③ It is a bird's-eye view. ④ It is the view around points P , Q from the left. Marks are leaning to one side. Only mark ab is perpendicular to the edge. It is quite different from paper-made Möbius strip. ⑤ Points on the edges and center line of the rectangular strip ① and Möbius strip correspond each other. But the length of their marked lines are quite different except line ab . So we shifted edges of the rectangular strip to right and left as shown. Now we possibly say that the length and angles of marked lines on the rectangular strip ④ and Möbius strip are identical. However, if we make a paper-made Möbius strip with this kind of parallelogram, it will have no smooth edge but a cusp at the joint. So to actually construct a Möbius band a parallelogram will not do. Instead we need a trapezoid, as originally proposed by Möbius himself.

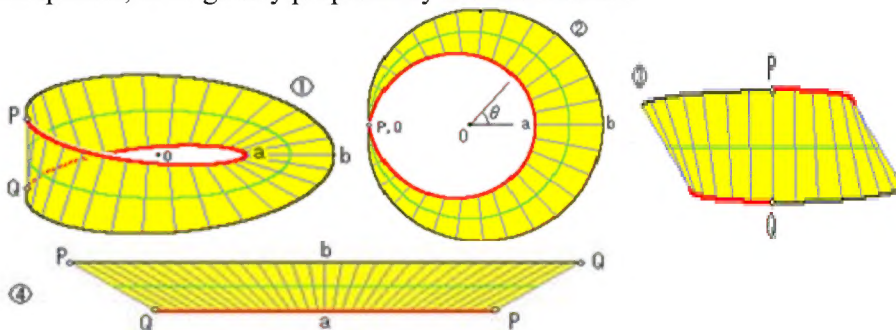


Figure 7.

This time, first we draw Möbius strip, and then we cut and open it as a rectangular strip. ① The green center line is a circle. Its center is O . Every notch line is perpendicular to the green center line at regular intervals of n division which is similar to [1]. Notches may look coarse but the number of them is the same as that of [1]. ② It is a bird's-eye view. θ is the angle of notches. See In [1] lengths of the red edge and black edge are the same as well as the rectangular strip. But in [2] it isn't. In the figure the length of black line is one and a half times as long as the red line. ③ It is the view around points P , Q from the left. Nick PQ is perpendicular to the edge. So is ab . ④ It is what we cut and opened Möbius strip while keeping the length of edge with notches. It is formed into a trapezoid and the length and slope of notches lines are distorted. This distortion is of great physical import. Indeed, while the fundamental rule of the sum of vectors, say forces, on Euclidean space is valid in the case of the parallelogram, in the case of Möbius strip, i.e. the trapezoid, this is no longer the case. In fact, this is shared by spaces with torsion, in which parallelograms do not close so that the sum of two vectors does not yield the diagonal, because the diagonal itself is ill-defined. Thus, instead in the non-closure of the parallelogram formed by the vectors u_P and v_P at P and their parallel transport, u_R at R and v_Q at Q , a fifth side $T(u,v)$ closes the infinitesimal vectors.

Prolonging $T(u,v)$ to the line joining P and Q , we have the trapezoid whose completion is depicted in red.

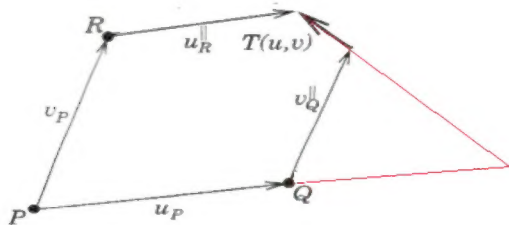
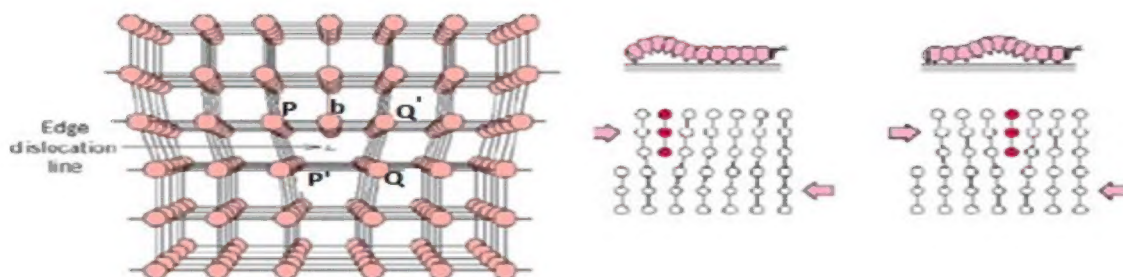


Figure 8. The infinitesimal parallelogram of vectors drawn in black does not close, the torsion being the fifth side closing the otherwise open parallelogram. We prolong it to draw in red the trapezoid naturally introduced by torsion.

3. The self-referential torsion geometry



Observe figure 9 above. We introduce torsion by shear in a lattice; in the rhs by a caterpillar moving in the surface of a perfect crystal. In the lhs we have a perfect lattice, as is the case of a discrete space-time rendering of the homogeneous spacetime of General Relativity, but in the central area in which a dislocation is the case. We see then the transition from parallelograms that close (null torsion as in General Relativity), to non-closing and the formation of a fifth side, the torsion field. Figure 9 shows the meaning of torsion. In an otherwise perfect crystal (whose vertices are described in red), i.e. one free of inhomogeneities (in practice, difficult to achieve) an edge dislocation is produced, either by removal (i.e., introduction of singularities) of atoms of the crystal, as the figure shows, or by introducing extra material, in short, both inhomogeneities. Torsion can be introduced by shear (i.e. the relative motion of two planes) as figure 9 shows. Think of a caterpillar which moves a lattice a step at a time, and the shear produces the torsion of the crystal; this shear produces a vortical motion on the vertical plane to the shearing plane under the mixing of layers for small that in can be. Another analogy is that of a rug, which moving in the perfect background of the homogeneous crystal; local changes affect the whole structure; the analogy strikingly applies to the crease (the folded rug) formation in the gastropore invagination in Embryology [9,10]. It can also be produced by a hole in the surface, producing an embryological expansion wave. Thus, it is an action-dependent participative geometry introduced in terms of inhomogeneities by the subject, say the caterpillar, or more basically the photon. This stands in stark contrast with the homogeneous situation of a Cartesian ideal geometry exterior to the subject, which corresponds in the continuum limit in which the atoms of the crystal approach indefinitely, i.e. the continuum hypothesis of Einstein, which due to the lack of a singularity, it corresponds to the zero torsion metric-based geometry of General Relativity. In short, to have loci, self-referentially dislocations are needed; these are the inhomogeneities that make a geometrical locus. The most basic dislocation is produced by a photon, a quantum particle, a singularity of an electromagnetic field. The parallelograms where inhomogeneities are present do not close, while in the perfect crystal do close, indicating a self-referential trivial loop: by default is indeed a loop, i.e. closed without mediations. Instead, in the former case a pentagon is produced. The

fifth newborn side at the upper right side of the center, joining Q' with b , is the torsion –the self-referential mediator; it is necessary and sufficient to the effect of completing the self-referential closure of the otherwise closed parallelogram; we shall explain this further below. As figure 9 shows in the center, the torsion appears producing a trapezoid with vertices P , Q , P' and Q' , since the upper side is now longer than the lower one. Yet trapezoids are themselves Möbius strips cut transversally, say along the line on the Möbius strip below joining P (P') and Q (Q') below, and laid open on a plane, which by identifying the opposite lateral sides PP' , QQ' , on a previously 180° twist of the trapezoid, re-establishes thus the Möbius strip, as shown below in figure 10.

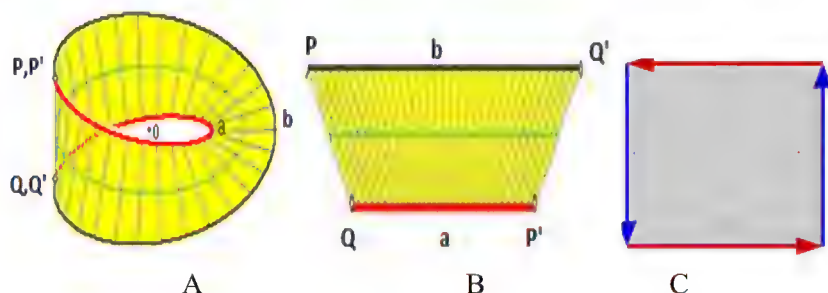


Figure 10: The Möbius band cut along PQ unfolds to the trapezoid in the middle with P on the Möbius strip connected to $Q' = Q$ (the upper side), and Q to $P' = P$, the lower side, respectively. On the rhs, C depicts the identifications that produce the projective plane, a non-orientable surface.. This completion (actually a self-referential mediator), the torsion, establishes a loop, say starting on Q goes to P' , continues to Q' , further to P via b (changing thus the direction of this side contrarily to the direction of the shear) to finally reach back to Q ; we depicted these directions on Figure 10C. Thus, it is a non-trivial self-referential action, to return to the identity, either on the sheared plane or out of it by the vortical twist installing the Möbius strip, depicted by the blue vectors in Figure 10C. As a loop yet with its upper and lower sides (drawn with red) now being considered as the edges of the Möbius strip, it establishes the characteristic uniqueness of edge of the Möbius strip, since the seemingly distinct edges PbQ' and QaP' , which are transversed with opposite directions relative to the shearing motion, become parts of a single cycle; otherwise stated, this is the 2:1 harmonic. Thus, would we finally identify them topologically with their opposite directions as in Figure 10C with sides drawn on red, we get the projective plane, rather than the Möbius strip. In the case of Einstein's General Relativity, the closedness of the loop is by default, since zero shear is assumed and thus, the torsion $Q'b$ does not appear if not the null vector, and thus the trapezoid collapses to a parallelogram; this is the case of the metric derived Levi-Civita symmetric connection of General Relativity, and the essential case is geodesic motion; yet, it is the spin coupling to linear momentum what produces a modification of this inertial motion on considering a connection with non-null torsion [61]. Would the fifth side $Q'b$, be equal in length to the other four, we would have the five-fold symmetry typical of quasicrystals and thus the Fibonacci sequence is embedded in this symmetry and a fortiori, the Golden Mean, Φ . As a final comment, while Fock noticed that in its dependence on metrics and covariant transformations of mathematical expressions on terms of which Einstein formulated his Theory of Gravitation had left unconsidered the problem of spacetime being uniform or not (see page 6 [111]), we clearly see that its implicit claim was that of uniformity.

4. Hyperbolicity and nonorientability of the Möbius strip

Hyperbolic geometry was introduced by Lobachevsky to lift the Euclidean axiom that parallel lines do not meet, as we already saw in the definition of torsion, the parallel transport of two vectors will not produce a parallelogram, so without any need of taking Lobachevski stance, parallel lines will intersect if the rule for parallel transport, i.e. the connection with torsion, is the case. Yet, it is related to projective geometry, in which appears related to lines at infinity. We want to dispel the ambiguity and elusiveness of this notion of infinity, relating it with the Möbius strip and the fact that infinity, as currently used by mathematicians and physicists alike, and no less to philosophers is a figure of

speech, that can be found to have a more specific and yet universal sense than that of the figure of an allegory of an unbounded growth. In mathematics and physics in which appears in various guises, as divergences, the idea of a transcendental unknowable imaginary world somewhat related to the material world of physics and everyday, in the continuity of real numbers, in particular in the non-denumerability and non-compactness of them, and still in the issue of non-computability that has arisen in such diverse fields as complexity theory, consciousness studies, etc. [109].

To do this, we start with projective geometry and the so-called gnomonic projection which is the fundamental operation for its construction.

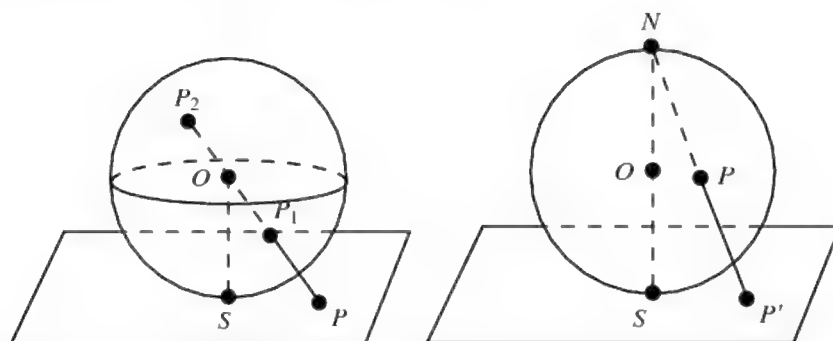


Figure 11: The gnomonic and stereographic projection of projective geometry and complex analysis, respectively.

We already introduced the gnomonic projection in figure 1. The gnomonic projection is a nonconformal map projection obtained by projecting point P_1 or P_2 on the surface of sphere from a sphere's center O to point P in a plane that is tangent to a point S . In the above figures, S is the south pole, but for the gnomonic projection can be any point on the sphere. Since this projection obviously sends antipodal points P_1 or P_2 to the same point P in the plane, it can only be used to project one hemisphere at a time. This is the why in Figures 1 and 2, we have also used the lower hemisphere. In a gnomonic projection, great circles are mapped to straight lines as shown in figure 1. The gnomonic projection represents the image formed by a spherical lens, and is sometimes known as the rectilinear projection. It is to be distinguished from the (conformal) stereographic projection of the Riemann sphere, in which the point of projection is the North Pole replacing the center O , and the contact point with the surface can no longer be arbitrary but the South Pole, identified in the complex plane S as the origin of the complex numbers. The naming of the Gnomonic projection comes from the Greek gnomon, the pedestal of a sundial (solar clock), which indeed it is not placed on the centre of Earth but on any arbitrary point of its surface, and by which the shade of the Sun projecting on a surface would be marked to identify the time of the day. It was this projection which was used not only to keep track of the time of the day, but the tracking of the seasons due to the precession of the equinoxes derived from the wobbling of Earth's inclined axis further moving along the circumsolar elliptic orbit. The overall graph of this annual projection is the Analemma, the Möbius strip projected on the surface of Earth.

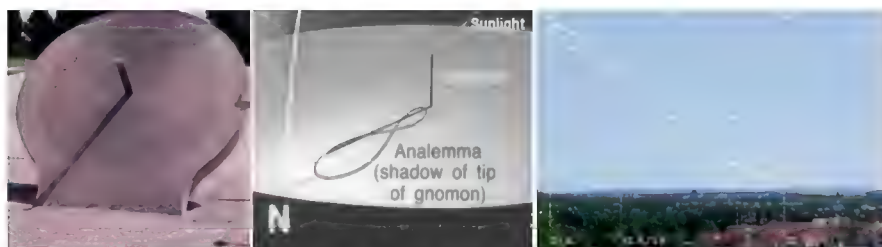


Figure 12: On the lhs a sundial produced by the shade of the Sun's projection on the gnomon, placed at the center of the device. In the middle, we see the figure 8 of infinity (the Analemma), produced by

the circumsolar rotation of Earth due to the wobbling tilted axis of rotation, on the elliptic orbit around the Sun. In the rhs the superposed annual photographs of this effect as captured by a camera lens placed in a fixed point and in a fixed direction, as in the rhs of Figure 12.

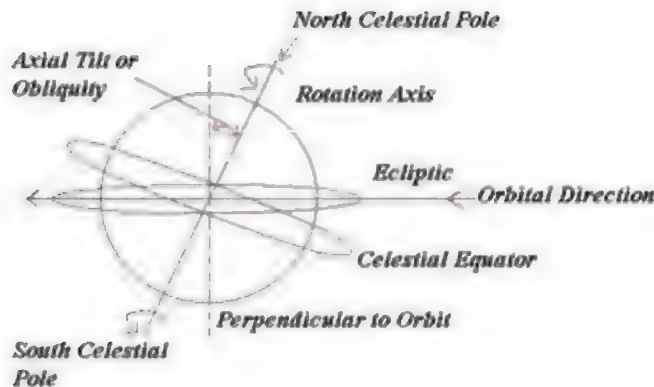


Fig. 13: The precession of the equinoxes produced by the circumsolar tilted wobble of the Earth's axis along the elliptic path of the centre of Earth; the angle of tilt and the eccentricity of the orbit are periodic variables, crucial to geophysics.

We return to the gnomonic projection to introduce the issue of infinity in projective geometry and the Möbius strip.

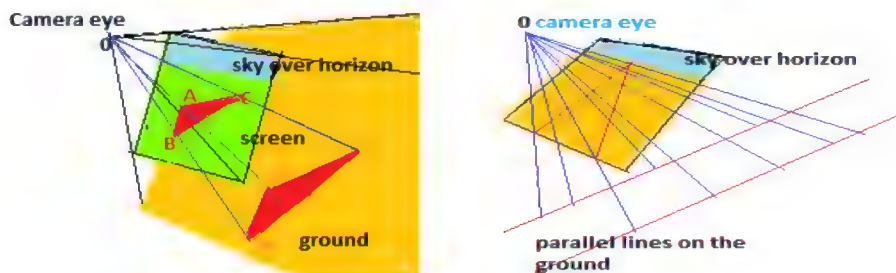


Figure 14: The gnomonic projection and the line at infinity.

So we imagine the projection of a point 0 , which now places the role of the centre of the sphere which we remove and still we take for the surface on which we practice the projection, to be a section of the tangent plane to Earth's surface.. The camera is the projection center and takes the picture of triangle $A'B'C'$ on the ground. This ground is supposed to be the tangent plane. The light green area is the view from the camera. Black rays from the camera indicate stereo area (pyramid) of the viewfinder. The square board fitted to black rays is to show what we can look at through the camera. Let us call the square board the -screen. The photo of triangle $A'B'C'$ on the ground is taken as triangle ABC on the-screen. Blue rays are projective lines of vertices of triangles. Consider now a pair of parallel lines on the surface and we lift them as if derived from the gnomonic projection of two lines drawn in red. These lines do intersect, in fact they do so on the horizon, on which we place the vertex/intersection point, with the horizon drawn in light green and the sky in light blue. Thus, the horizon which in projective geometry is called the line at infinity is not such line in the actual world in which the tangent plane ceases to provide a good approximation to the surface of the sphere. It is neither an imaginary line, but in the definition of mathematicians, as we already saw in identifying the line at infinity in the gnomonic projection as the equator of the sphere. Instead of the elusive imaginary line at infinity, we can identify it in the panel screen as the boundary between Earth and sky. It is no line at infinity at all. It can be thought as the manifestation of the non-local embedding of the Möbius strip on the projective plane, as we discussed at the beginning.

So again, what physicists would name a divergence, has a very concrete manifestation, and the very notion of infiniteness is called into question. Of course, we can take the camera's eye for the North Pole instead of its substitution for the center of the sphere, and the conclusions still apply to the

stereographic projection; thus the conclusions do not require we have a conformal (stereographic) nor a non-conformal (gnomonic) projection. In fact, for visual perception, in which the centre of the retina plays the role of the point of organization of incoming light, it is the conformal mapping that applies, as we shall discuss below. We have already seen that the antipodal identification that is the case of the gnomonic projection, produces the Möbius strip, in which the line at infinity is precisely the line on which the Möbius strip is produced by the twisted identification of antipodals along a section of the equator/line-at-infinity. Let us see how is this further related to the actual asymptotic hyperbolic discontinuities on the real plane, which as we shall see, become continuous in the Riemann sphere by introducing the one-point compactification, and it is related to the non-orientable Möbius strip structure of the complex plane itself, which in the case of visual perception, it is related to the complex logarithmic map as defined on a Möbius strip instead of its usual orientable domain.

Consider the gnomonic projection now applied to an hyperbola placed on either a screen or on the projective plane as in figure 15.2 with the screen placed on top of the equator, which is the line at infinity; for the latter note that since the asymptotes all end on the line at the equator, this produces a compactification in which instead of discontinuity at the origin, we have a continuous curve so that the real positive and negative infinities are identified together with the positive and negative imaginary infinities.

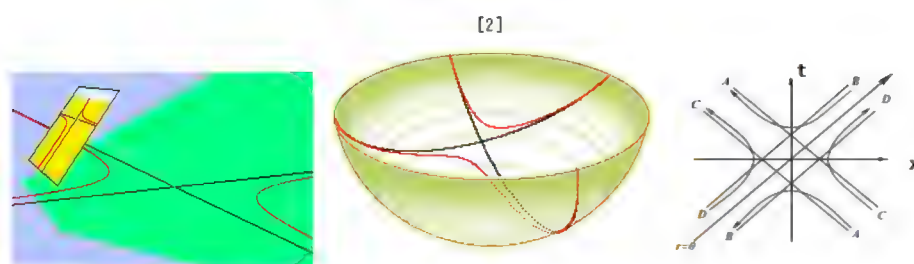


Figure 15. The projection of an hyperbola on the ground which in the case of the figure on the middle, rather than a discontinuity on both the real and imaginary axis, they are continuous via the line at infinity, which is the equator of the sphere, still identified by antipodal points. Right: we have the hyperbolic trajectories associated to the closed time loops in the Gödel solution to Einstein's equation (modified from [58]).

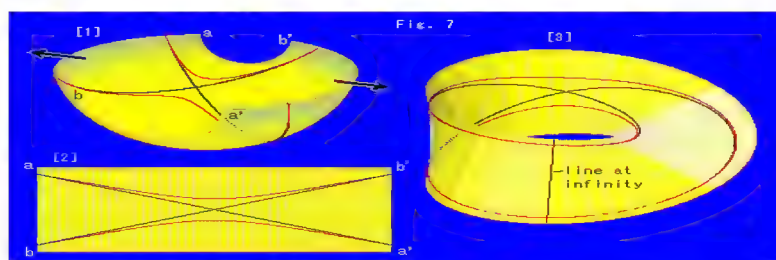


Figure 16 above is to see a full range of a hyperbola placed on Möbius strip. A part of projective plane is the Möbius strip. [1]: Cut the hemisphere in Fig. 16 straightly down as shown. [2]: Stretch it in the arrow direction by force, and make a belt. [3]: Twist one end (right or left) of it half, and glue it to the other, a to a' and b to b' . Then we get a Möbius strip with the hyperbola that was in [1]. The jointed line is the straight line at infinity. The red hyperbola is a double-turned loop now. Each asymptote forms a loop and meets with the hyperbola on the line at infinity.

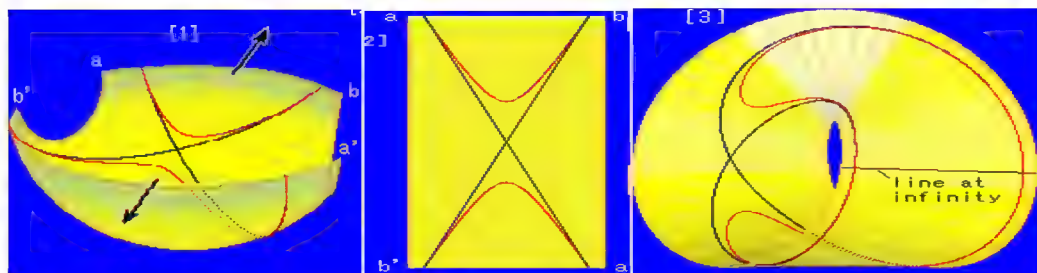


Fig. 17 above, is also to see full range of the hyperbola on the Möbius strip. The hyperbola is the same as that of Fig. 16 but we cut the hemisphere in another way. [1]: This time, we cut the hemisphere in another direction as shown. [2]: Stretch it in the arrow direction by force, and make a belt. [3]: Twist one end (top or bottom) of it half, just as we did before, and we get a Möbius strip. This time the red hyperbola is not a double-turned loop but a single loop; we shall return to this issue with regards to closed geodesics in General Relativity. It is because that Möbius strip is not the entire projective plane.

5. On turning inside out, the Klein bottle surface and the 2:1 resonance

We introduce the Klein bottle by producing the identifications depicted in the next figure:

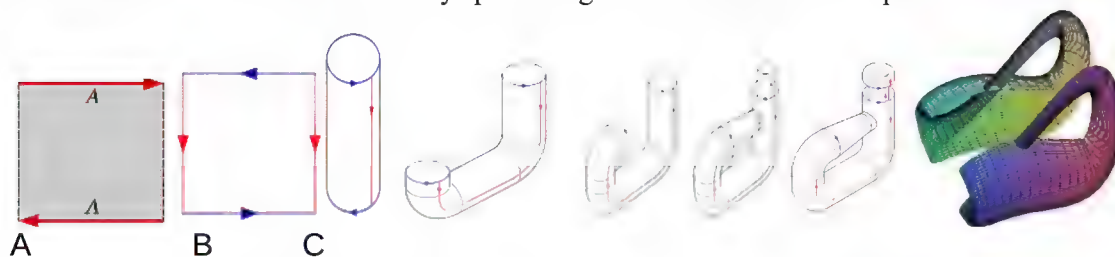


Figure 18: The identifications as they unfold to produce in A the Möbius strip and in B the Klein Bottle surface; courtesy of Inductiveload . C depicts the folding sequence to immerse the Klein Bottle in 3D. In the final picture –courtesy of Theon . we see two oppositely twisted MBs produced by cutting the Klein Bottle along the longitudinal section; conversely, zipping them we obtain the KB. Yet a single MB can be obtained from the KB



Figure 19: A natural logic of 4 states (the Klein Bottle Logic, KBL) which has two digits representation: Inside-Inside (represented as 11), Inside-Outside (represented by 10), Outside-Inside (represented by 01) and Outside-Outside (represented by 00). The mediation states arise from self-penetration, absent in the mechanical-dual-membrane and in the Cartesian conception of object-in-space-before-subject, are paradoxical states associated to time waves. From the KBL appears the genetic code with its 64 elementary codons, and furthermore the genome and its association to the mathematical structure of the Dirac algebra of Quantum Mechanics [12].

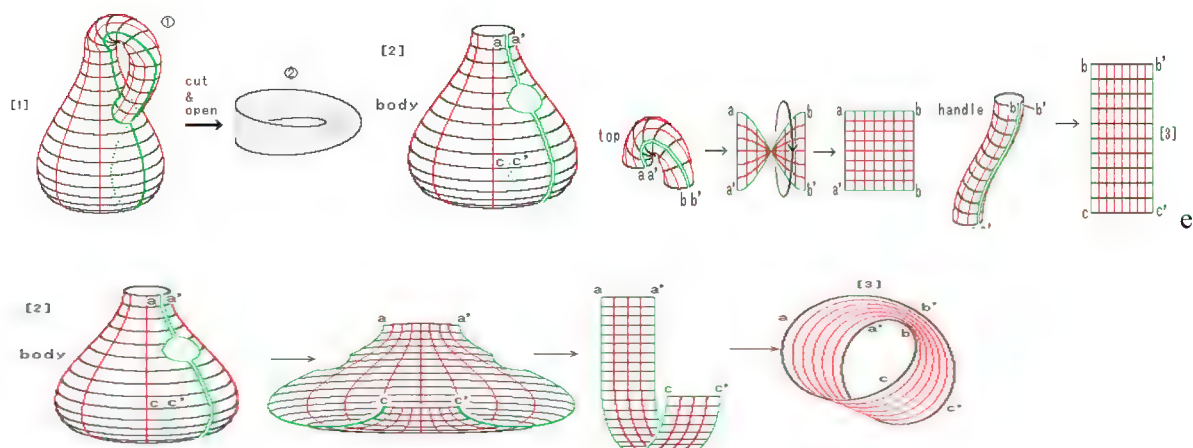


Figure 20: Surgery sequence to extract a single Möbius strip from the Klein Bottle.

Yet, by appropriate surgery on manifolds, we can find that a single Möbius strip lies on the surface of the Klein bottle. Indeed, consider [1]: We cut Klein Bottle ① along the green line. Then we get Möbius strip [2]: In order to see how the single Möbius strip comes out, we separate Klein Bottle with green cut line into "body", "top" and "handle". And we shape them into rectangles. Pay attention to that the cut line on the top is twisted by 180° . So we untwist it on the way as shown with the arrow.[3]: Three rectangles are rejoin, and we get Möbius strip. The rectangle of the top was untwisted but what rejoin is automatically twisted again .[4]: It is the series of squares with arrows that represent the cutting above. The sequence of surgeries is represented by the figure below:

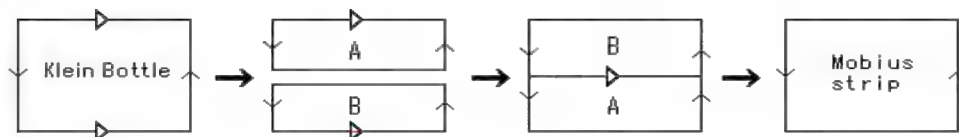


Figure 21: Schematic surgery of the Klein Bottle for extraction of a single Möbius strip.

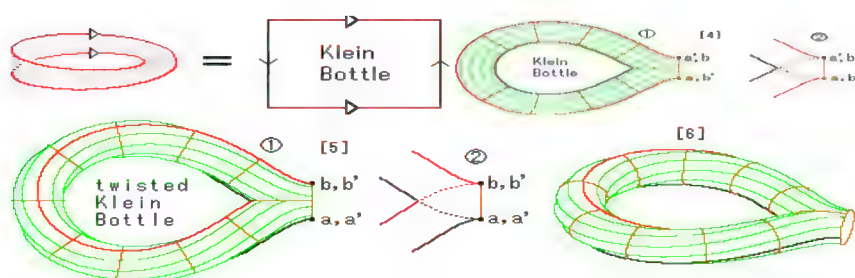


Figure 22. Constructing the Klein Bottle from the Möbius strip. [1]:The square with arrows ① represents Klein bottle. We begin with it. First we make a cylinder ② by gluing delta arrows. The red lines are just for illustrations.[2]: We try to make a torus-like, but the arrows never be in the same direction whatever we twist the tube. It is a well known matter. [3]: But bend the tube in U-shape, and the arrows are automatically arranged in order without twisting. [4]: Suppose the surface of U-tube is free to go through, and we joint both ends of U-tube while keeping their arrow directions unchanged as shown ①. ② is a transparent view around the joint. The shape is completely symmetric. [5]: The Klein bottle is made for an appearance. But it is not ready to correspond to Möbius strip of [1]. So we must twist the tube by

180° though arrows are already identified. ① is twisted one. Compare the transparent view ② with that of [4].[6]: It is a stereoscopic picture of [5]. Observe the black curve in [6]: which if continued another 360° turn it turns to be the red line of the top of the figure to return to an original point along any such green, red, or black curve. Here we find the topological genesis of the 720° rotation of the neutron submitted to electromagnetic field, or still, of the double covering of the Lorentz group which is the symmetry of the Dirac equation

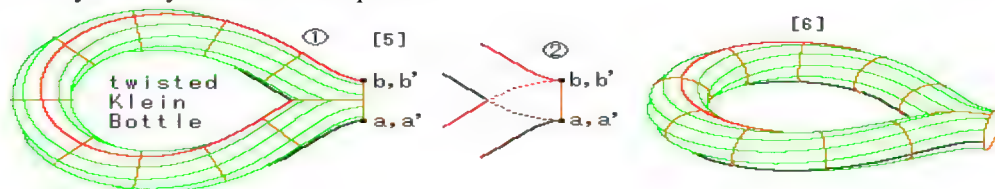


Figure 23: twisted Klein bottle

This also is the fundamental 2:1 resonance of celestial mechanics and of music, to be discussed below. Its importance to physics and astronomy, seems to have escaped the attention of Möbius, an outstanding astronomer. Indeed, consider the figure

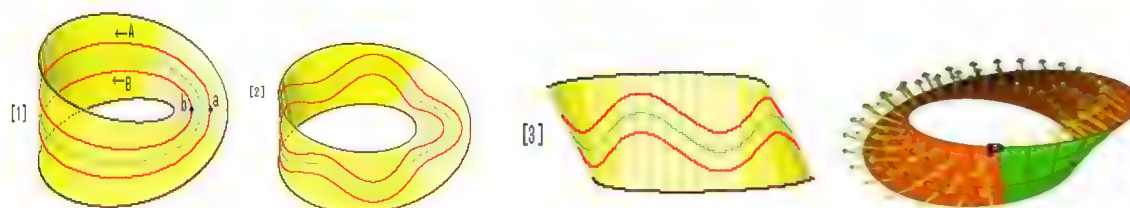


Figure 24: Left: the 2:1 harmonic of the Möbius strip. Right: the protoform of Newton's Third Law. Point P' lies on top P , on the other side, modulo 360° turn.

[1]: The green line is a center line. Suppose the red line extends counterclockwise from point a . And it turns back and arrives at point b which is the opposite side of point, so to return to b again another complete 360° turn is required. The same is the case of the red line extends one more round, it gets back to the started point a . The red line never went across the center line. Nevertheless the red line runs both side of the center line. The red line looks like parallel lines when we see them partially. But it is not true. Route A and route B are actually a single route. The green line looks a median or a central reservation on a road but it does not divide Möbius strip into two. If we cut Möbius strip along the green line, its length becomes double. There is no opposite lane on Möbius strip. This is the topological origin of the 2:1 harmonic. It is also the origin of a protoform of Newton's Third Law, albeit it does not require, in distinction with Newton's formulation, a dualistic assumption, nor the instantaneous causality that this law implicitly assumes; causality, if any, is embodied by the 2:1 resonance produced by the 180° twist. Indeed, consider a normal vector to Möbius strip; if we move it along any curve in a 360° turn as before, we would find it pointing in the opposite direction, and equal in its length, without the need of an assumption as in the Third Law. Another 360° turn will return the vector to the same point and to coincide with its original configuration. Rather than having an action and a reaction, in the Möbius strip and in the Klein Bottle, the opposite and equal modulus of normal vectors is a resultant of the 2:1 harmonic, not an hypothesis for the foundations of physics at large. So it appears that this harmonic is more fundamental to physics than Newton's Third Law, which invokes an instantaneous symmetric causality. As we said before, this is the case of the origin of the 4π symmetry of relativistic quantum mechanics, and shows up in the precession of the neutron submitted to an electromagnetic field [60].

Interestingly, resonances –say in celestial mechanics and chaotic systems- are discussed in terms of the 2-torus [110], so we want to relate the torus to the Möbius strip and the Klein Bottle to the 2:1

resonance; the latter is already the case, say, of the Kirkwood gaps for Jupiter's periods in relation with the asteroid gaps. The relation is that the 2-torus is the double covering manifold of both surfaces. Yet we want to illustrate the meaning of this in terms of the 2:1 harmonic. This jumps to the eye on viewing either Figure 24 or 49. It can also be seen in examining a 2-torus, which we assume to be solid, yet the 2:1 harmonic is already apparent in Figure 25 [1] below (courtesy of T. Ito):

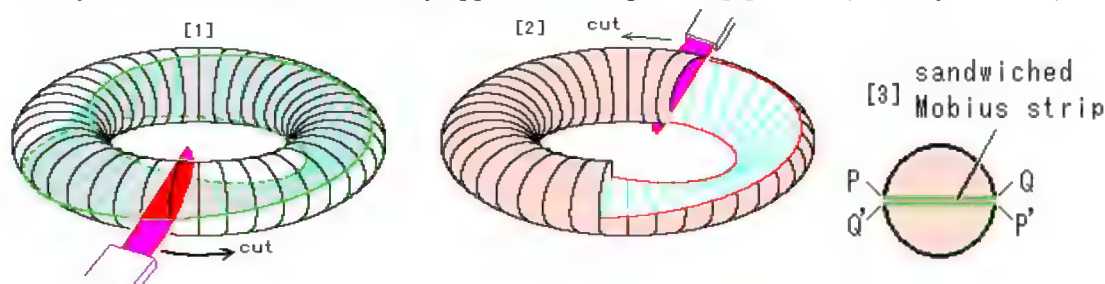


Figure 25: Cutting out the Möbius strip from the 2-torus by following the 2:1 harmonic

[1]: Möbius strip is lurking in the solid torus. We cut the solid torus along the green curve by the red knife as shown with an arrow. But the solid torus is not be separated into two. To get Möbius strip physically we have to scrape or melt the solid torus away. [2]: It is on the way of cutting. We are doing it while throwing unwanted parts off. We will be able to get Möbius strip if we continuously cut the torus by two circuits. [3]: We cut the solid torus with red knife. It is completed by two circuits. If we start to cut from the upper part PQ , the red knife will come to lower side $P'Q'$ at a circuit, so that we have to continue cutting one more turn. That is why the length of new torus is double. During full cutting, the red knife screws 360° . This 360° is what we observed on the surface of Möbius strip. The observation direction changes every moment while traveling. And the red knife itself turns around the hole of Möbius strip two times. It is similar to that of Earth circumsolar year rotation on an elliptic path, as it rotates precessing along its axis; this is crucial rotation around its axis, when submitted to an electromagnetic field.

Returning to the surgery of extracting Möbius strip from the 2-torus,

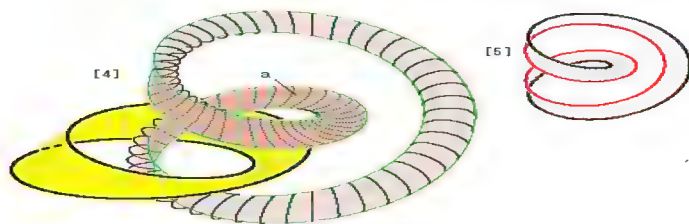


Figure 26: separation of the cutting of the Möbius strip from the 2-torus by following the 2:1 resonance.

[4]: Suppose the solid torus is rubber made. And we cut it without throwing anything off. We get the yellow Möbius strip and a long loop. The long loop is chained to Möbius strip and its length is double of Möbius strip. The dark surface a of the long loop was in contact with Möbius strip. We cannot get Möbius strip alone unless we cut off the long loop depicted shortened in [3]: Suppose the solid torus is rubber made. And we cut it without throwing anything off. We get the yellow Möbius strip and a long loop. The long loop is chained to Möbius strip and its length is double of that of the Möbius strip. The dark surface a of the long loop was in contact with Möbius strip. We can not get Möbius strip alone unless we cut off the long loop. [3]: Suppose the solid torus is rubber made. And we cut it without throwing anything off. We get the yellow Möbius strip and a long loop. The long loop is chained to Möbius strip and its length is double of Möbius strip. The dark surface a of the long loop was in

contact with Möbius strip. We can not get Möbius strip alone unless we cut off the long loop, depicted shortened in [5].

Therefore, to extract the Möbius strip from the 2-torus, we have to cut out the double length loop that is produced by the 4π rotation. In other words, the 2:1 harmonic is revealed as if embedded in the 2-torus by erasing its trace as an inherent rotation, that is neither an internal nor an external symmetry, the latter issue being another of the dualities that physics is used to claim to be fundamental. Of course, any astronomical body is extended, so the elliptic circumsolar motion is that of the centre, whose extended motion is indeed that of a solid 2-torus with the 2:1 built-in resonance.

6. Symmetries on the Möbius Strip and non-orientability in chemistry and biomembranes

We are lead to examine what are the inherent symmetries of the Möbius strip and the Klein Bottle, which we shall do next

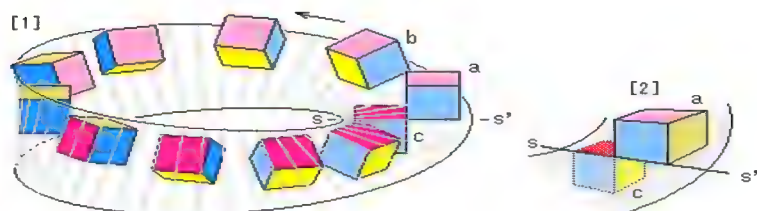


Figure 27: Representation of differentiated structures on a Möbius strip (courtesy of T. Ito).

Consider Figure 27: [1]: We slide a cube on the Möbius strip. It is coloured in light and shade of red, blue and yellow. If one side is light, its opposite side is shade. Cube **a** is at the start line **ss'**. We move it while keeping the shade red face is always on Möbius strip. Cube **b** on the way is showing the three of light colors. [2]: Cube **c** that has traveled a circuit and now at the start line **ss'** again. Compared with cube **a**, it is upside down, right side left, and shifted to **s** under Möbius strip. In chemistry the issue appears, as we already said related to enantiomerism, the existence of chemical isomers, which have a symmetry which is non-specular but implies a change of chirality.

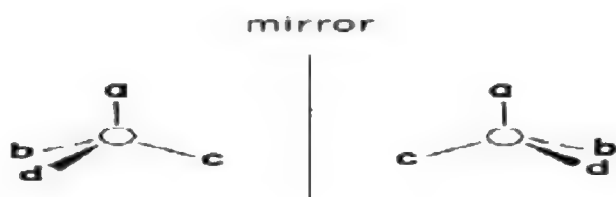


Figure 28: 2D representation of enantiomers in organic chemistry, after Emil Fischer.

The notation in Fig. 28 is the Emil Fischer' representation that chemists use for describing three-dimensional structures on 2D: a solid line in the plane of the paper; a dashed line points in back of that plane; a wedge to the front. The molecule at left is not identical to the one at right; if we superpose **a** and **d**, **b** and **c** won't fit; thus, they are called chiral molecules, since the hand descriptor of a thumb, pinkie, palm and back, play the same role as **a**, **b**, **c** and **d**., the chemical groups that differentiate enantiomers. The rotation of a three dimensional structure on the Möbius strip describes precisely enantiomers. This is crucial to the biochemical activity of molecules, and in particular, of proteins [37]; we recall that Pasteur discovered the chirality of molecules, which lead him to propose the complementarity of form and function in biology, which is known today as the Pasteur-Curie Principle, in recognition of Pierre Curie's work on the subject. Yet, what this examples teach us is that

the issue of chirality, in general, can be associated with the non-orientability of the Möbius strip, or still of the Klein Bottle.. There has been developed in the last years, a new paradigm of chemistry, and in particular of organic chemistry, in which non-orientable topologies are the very basis of the subject [38]. We here recall the cyclical structure of benzene.

Yet, the fundamental issue of turning inside out the charge configuration which is basic to the stereochemistry of molecules, is the basis for understanding this paradigm. In an early review to this topological paradigm for chemistry [38], it was claimed that "...The examination of topological structures is not only intrinsically of great cognitive interest, in the sense of the opportunities for realising mathematical entities in a material, molecular form. It is no less significant that the actual properties of molecular systems may cause behaviour which is topologically indeterminate but nevertheless almost the same as for a topologically ideal system." We notice that this is essentially the same thesis we have claimed for biology. and for systems in general [9,10,11,12,14,15]. One of the forms of topological isomerism deals with the same issue that cells do, the Outside and Inside which arises from the real non-pointlike character of atoms, namely, the set of spheres formed by the van der Waals radii of chemically non-bonded atoms in a molecule effectively create a closed surface. As a consequence, an atom or a molecule in the inner region is unable to pass into the outer region, since it is of finite size. Here we find at an atomic level the issue of penetration of biomembranes. The proposed topological hypothesis makes no demands on the size of the species or molecules to be transported (the migration of large molecules surpassing membranes are known. The main idea is that, at least in some cases, no real penetration takes place through the membrane (or the charge configuration in the case of a molecule), which is a one-sided non-orientable surface. The suggestion in this review is that if a particle Z is firmly attached to a site on the Outside of the membrane (or the charge configuration), induces a change in the conformation of the surface due to its bonding or some other cause, the site may move 'inside' in the sense of a Klein Bottle invagination, to wit: Z moves from the Outside-Outside through Outside-Inside and Inside-Outside to rest in Inside-Inside, as in Fig. 19. Thus the apparent result is the penetration of Z through the membrane, which is actually a consequence of the conformational change in the surface which becomes non-orientable—a special kind of the allosteric effect, which is conceptually predominant in biology. Biophotons emission due to change of conformation by re-organization of chemical bonding is known in biology as chemiluminescence [42,48] ; it is to be noted that chemiluminescence is recognized to be the basic process for syntropic (in distinction of entropic) organization of organisms using stored energy of chemical machines, and that is the resonant energy communication of quantum entanglement in organisms as argued in [39,40,42]. Most remarkable is the fact that the Klein Bottle logic which includes quantum, fuzzy and Boolean logics as subcases, is essentially an entanglement topologic due to the non-orientability of the Klein Bottle, so that the quantum coherence of living organisms has in this logic a logophysical operator and in biochemiluminescence a manifestation of quantum (topo)logical entanglement associated to stereochemical changes, which we have associated with the tensegrity structure of the cell's matrix associate to the tensegrity structure of the cell's matrix [10]. Remarkably, a molecule that can switch both topologies, orientable and Möbius strip, without breaking any chemical bonds, has been devised [41]. The Klein bottle topology has been revealed in Inorganic Chemistry [43], while the analysis of 'simple' molecular conformational data reveals that cyclo-octane rings have a Klein Bottle topology. Furthermore, the issue of non-orientable stereochemistries is the core of aromatics [44]. Sokolov's prophecy appears to be in the path of realization; also, the genetic code is based on the Klein Bottle Logic [12], the codons being the 3-fold tensor product of the four states of figure 19 resulting in the codons and anticodons lying one-to-one in either local side of a Klein Bottle which is further recurrent to a HyperKlein Bottle [9,12]. Therefore, we find already in stereochemistry, the logophysics we are claiming for biology. Thus, we suggest that it may be possible that bindings of molecules at the membrane can produce conformational changes without chemical bonds breakings, that transform its topology from two-sided orientable to one-sided non-orientable, as well as that of the molecule, with the production of bioluminescence. Yet, the interior of the membrane as a vacuous interior to a non-orientable bilayer

lipid would be identical to a massless quantum surface interface that appears in the Klein Bottle structure of the Periodic Table of elements [13], and thus associated itself to a boson field which we associated to the torsion photon field [15,19]; we shall retrieve this issue further below in examining the wavefronts of Earth's crustal tidal waves.

Alternatively, we could think of the bilayered cell membrane undergoing a stereochemical transformation induced by adhesion to its boundaries of integrin molecules [46]. Thus, would a stereochemical transformation through binding at the surface of the membrane occur, it would be accompanied with photon emission of both the membrane and the adhered molecule, which would produce an electromagnetic signal that would lead to non-local effects both Exterior and Interiorwise, which under the topological change from orientability to non-orientability, would lose the dual distinction. The extracellular matrix as a tensegrity (i.e. in which tension is continuously distributed to the whole –think of the spider's web) structure. This matrix is connected with the cytoskeleton –through molecules known as integrins in adhesion sites of the membrane [46] which we proposed to modify the membrane's topology; this holonomic system: connective tissue, cytoskeleton, nuclear matrix, has been termed the living matrix, and it is known that by the piezoelectric effect, electromagnetic fields are produced leading to instant signalling across the organism [47]. Hence, it is natural to conceive, unless we introduce the Exterior & Interior duality, that the tensegrity structure for the living matrix has the Klein bottle logic. This matrix does not only carry energy, but also information, and quantum phenomena are at its roots. This radiation field might be associated with the mitogenetic field discovered by A.Gurwitsch [48], which lead to biophotonics as well as to the studies of the torsion phantom effect discovered by Gariaev [49]. In our conception, the photon is a universal objective-surjective gestalt, which cells have proved to be able to self-organize in terms of their 'perception'[50]; we can venture –following [50]- that the cell's tensegrity living matrix is the gestalt in question and that the signaling to which it responds are electromagnetic waves carriers of biophotons. We have identified in the bioresonance through biophotons, the source of selfhood [15], which is identified as the quantum coherence of living organisms in syntropic self-organization [39,40]. The tensegrity structure of cells under differentiation was associated to torsion electromagnetic waves responsible for the contraction and expansion waves that determine the tissue to result in the differentiation process [10]. In fact, due to the identity of wave-fronts of elasticity and electromagnetic fields, this is a tensegrity which is both ruled by classical and quantum physics, which we shall further encounter, in geophysics, further below.

7. The Klein bottle, two-dimensional signals and the embedding in four-dimensional space

The Klein bottle appears as a form of forms for matter, as is the hidden symmetry-topology of the Mendeleev table of periodic elements when the stable nuclides are included [13]. The Klein Bottle as a 2D surface requires self-penetration for its construction (for the relation of this with homotopy see [31]); thus the classical conception of space as a container for objects, which is the imperating paradigm in physics and in the sciences at large, does not apply to this surface. Instead, the classical (in the sense of space as a container for forms) response to this impossibility of constructing the Klein bottle if not by self-penetration, is claimed to be surmounted by embedding it in four dimensions, where the self-penetration can be avoided, as an abstract construction, albeit one that can not in practice be carried out!

In pattern recognition through artificial means, analyzing digital photographs, the Klein Bottle appears as a form of forms, and its identification does not require an actual 4D construction, but in fact, in the artificial case, the construction of a seven-dimensional space, actually a sphere, from which the barcodes of the images are revealed to hold the Klein Bottle as a universal pattern [27,28]. The topological structure of simple signals in 2D with arbitrary phase and orientation is described by a Klein Bottle [30,31,51]; here the notion of a signal is the one in use in the theory of communication and image processing, any function (to be encoded/decoded, transmitted, filtered, enhanced, decomposed in Fourier harmonics or wavelets, etc.); yet for what we have just stated, most remarkably

they appear as signals in the sense of wavefront discontinuities already discussed which we shall later elaborate.

Indeed, the issue is that *fringe* signals have a definite orientation (so here is where the boundary conditions come in) as opposed to a 1D signal, thus introducing a 2D phase, and thus the codification of such a signal turns out to be mathematically feasible in higher than 2 dimensions. Already given an image of dimension D ($D = 2$ for ordinary images), a mathematical representation of a local *intrinsically 1D* region (i.e. the number of dimensions necessary to describe the signal is one), say $f(\mathbf{x}) = g(\langle \mathbf{x}, \mathbf{n} \rangle)$, where f is the image intensity function which varies over a local image coordinate \mathbf{x} in \mathbb{R}^D , g is a one-variable function, \mathbf{n} is a unit vector and $\langle \mathbf{x}, \mathbf{n} \rangle$ is the inner product of \mathbf{x} and \mathbf{n} . The intensity function f is constant in all directions which are perpendicular to \mathbf{n} . Intuitively, the orientation of an D -dimensional region is therefore represented by the vector \mathbf{n} . However, for a given f , \mathbf{n} is not uniquely determined. If $\mathbf{n}' = -\mathbf{n}$, $g'(\mathbf{x}) = g(-\mathbf{x})$, then f can be written as $f(\mathbf{x}) = g'(\langle \mathbf{x}, \mathbf{n}' \rangle)$, which implies that for antipodals, $\mathbf{n}' = -\mathbf{n}$, it is a valid representation for the local orientation.



Figure 29. On the left, a non-unique representation for orientation of images on 2D for antipodals, except for a phase shift. On the right, the double angle representation of orientation (reproduced modified from [51]). This is similar to the case of music perception of the half-octave (tritone); see Fig. 57. This duplication of angle is nothing else than using the 2:1 harmonic inherent to the Möbius strip and the Klein bottle, in this case to be able to assign a unique local orientation to a 2D image.

The conditions for representation of higher dimensional signals is as follows. Three requirements for a phase representation in higher dimension must be fulfilled. Firstly, the signal neighborhood must be intrinsically 1-dimensional. Secondly, the phase representation must be continuous. As a consequence of the two first requirements, the direction of the phase must be included in the representation.

Consider a signal $f(\mathbf{x}) = a(\mathbf{x}) \cos(\theta(\mathbf{x}))$ depending on a 2D phase $\theta(\mathbf{x})$. Locally, the phase can be described by the linear model as a function of the offset $\Delta \mathbf{x}$ from \mathbf{x} ,

$$\theta(\Delta \mathbf{x}) \approx \theta + \langle \nabla \theta, \Delta \mathbf{x} \rangle = \theta + \rho \langle \mathbf{u}, \Delta \mathbf{x} \rangle, \quad (1)$$

where the local orientation has a direction, $\nabla \theta = \rho \mathbf{u}$. Let us here and henceforth assume that f is a sinusoid in 2D, i.e. the amplitude $a(\mathbf{x})$ is assumed to be one, and $\mathbf{u} = (\cos \varphi, \sin \varphi)$. The analytic signal for such a sinusoid takes the form (via the Riesz transform of f),

$$\mathbf{r} = \sin \theta (\cos \varphi, -\sin \varphi), \quad (2)$$

where we have fixed the amplitude a to be equal to one [51]. This vector is a sufficient representation of orientation for a fixed $\theta = 0$, but if θ varies problems occur. The amplitude of the signal, here equal is mixed up with the phase and for $\theta = 0$ the information about orientation is lost. Furthermore, the representation is not unique since identical signals can appear as antipodal points as seen in fig. 29. An elegant remedy to the latter problem is, in 2D, to compute the non-linear combination of the sinusoids, such that

$$\mathbf{r}(\phi) = \sin^2 \theta (\cos^2 \phi - \sin^2 \phi, 2 \sin \phi \cos \phi), \quad (3)$$

where $\mathbf{r}(\phi)$ rotates with twice the speed; this is known as the double angle representation. This is, of course, equivalent to the 2: 1 resonance.

A representation of the Klein Bottle must satisfy the following identities

$$\begin{aligned} \mathbf{r}(\phi, \theta) &= \mathbf{r}(\phi, \theta + 2\pi), \\ \mathbf{r}(\phi, \theta) &= \mathbf{r}(\phi + \pi, 2\pi - \theta). \end{aligned} \quad (4)$$

A Klein Bottle cannot be embedded without self-intersections in \mathbb{R}^3 so that the representation of fringe pattern requires to transport the self-penetration to a higher dimension in which a definite signal representation for orientation can be achieved: one adds dimensions to represent as if different, the superposition of singularities of the phase orientation as occurs in the cortical visual process (see figure no. which explains why the information of orientation vanishes in the phase representation and that simultaneous representation of orientation and phase could not be accomplished with only 3 elements. One possible representation of the Klein bottle in \mathbb{R}^4 is

$$\mathbf{r} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta (\cos^2 \phi - \sin^2 \phi), 2 \cos \theta \cos \phi \sin \phi). \quad (5)$$

Consider the intrinsically 1D signal $f(\mathbf{x}) = \cos(\rho \langle \mathbf{u}, \mathbf{x} \rangle)$ where \mathbf{u} , as previously, is defined by ϕ , and the vectors

$$\mathbf{r}^1 = (\sin \theta \cos \phi, \sin \theta \sin \phi), \mathbf{r}^2 = (\cos \theta \cos^2 \phi, \cos \theta \sin^2 \phi, 2 \cos \theta \cos \phi \sin \phi). \quad (6)$$

These vectors are the first and second Riesz transform of the 1D signal f [51]. Consider next the composed vector $\mathbf{r} = (\mathbf{r}^1, \mathbf{r}^2)$; it satisfies eq. (4). Thus, \mathbf{r} gives a simultaneous representation in \mathbb{R}^5 for both amplitude and orientation; the effect of going to a higher-dimensional representation is to do away the singularities that may appear in low dimensional, concretely in 2D as the paradigmatic case; we have already discussed this issue in connecting hyperbolic geometry to the re-entrance of the Möbius strip and we shall later discuss it in relation of the divergences that arise with geophysics. In fact the Klein Bottle representation in \mathbb{R}^4 is the projection onto the spherical harmonics of order 1 and 2 of the 1D signal $f(\mathbf{x}) = \cos(\rho \langle \mathbf{u}, \mathbf{x} \rangle)$ [51].

This is quite remarkable: The form of forms is nothing else than the projection to order 1 and 2 in the space of spherical harmonics of the sinusoidal 1D signal with vector orientation that arises from the fringes of intrinsically 1D images. Of course, this is not exclusive to images, but to any such sinusoidal signal, and specifically, that arises from the fact that the orientation can not be uniquely defined on the boundary of the domains under examination, so it is about multivaluedness and superposition, which is precisely the essence of the Klein Bottle Logic and its Matrix Logic representation, and one of its physical manifestation is the lack of well defined tangential directions of wavefronts of hyperbolic partial differential equations, to which we turn further below.

8. Hyperbolic spacetime, time loops and nonorientability

The relation of non-orientability to the hyperbolic geometry of General Relativity can be approached as follows. It is widely believed that spacetime must be both orientable and time-orientable. Supporting arguments are that there is no evidence of a lack of orientability and that a non-orientable spacetime would be incompatible with the observed violations of P (parity) and T (time reversal) invariance. Yet, these stringent conditions can be relaxed by noting that spacetime (modeled as a Lorentzian manifold, and thus hyperbolic metrics are the case) are locally orientable (as is the case of the Möbius strip, the Klein Bottle – away from the self-penetration – and the projective plane) while on a non-orientable spacetime, there exists a closed path along which a consistent orientation cannot be defined; this follows from the definition of orientability and topological argument. Furthermore, if spacetime is not orientable, then through every point there exists a path on which an orientation cannot be defined.

These issues are the core of what has come to be considered a revolution in cosmology, i.e. the possibility that rather than an uni (single)-verse, the case is that of a multiverse. The connection in the multiverse is produced by ringholes or wormholes, namely holes in spacetime connecting the inter-universe tunneling. This hypothesis requires the existence of other universes should actually require choosing spacetime holes which be (i) non orientable and (ii) convertible into a time machine with completely unspecified relative speed between its mouths. One such space-time construct has in fact been already studied, in the shape of a Klein Bottle hole [111]. In this case, non orientability is guaranteed by the existence of a throat with the topology of a Klein Bottle and it was also shown that this space-time can be stable to vacuum fluctuations and is also convertible into a time machine with fully arbitrary inter mouths speed. Lensing effects appear associated to the non-orientable nature of the connecting Klein Bottle throat. The metrics that support these connections are solutions of Einstein's equations.

We recall that Gödel introduced a solution of the Einstein equations with cosmological constant under the following conditions, as stated by Boeyens [58]: 1) Spacetime is homogeneous and has rotational symmetry. 2) (Neighbouring) world lines of matter are equidistant. 3) A positive direction of time can be introduced in the whole system. 4) The direction of the time flow is not uniquely defined for each spacetime point. 5) Every worldline of matter is infinitely long and never returns to the same point; but there also exist closed time-like lines. 6) There are no space-like three-spaces. 7) There is no absolute time. 8) Matter everywhere rotates relative to the compass of inertia. Gödel's solution has been widely ignored because it fails to predict a Doppler redshift, does not give a clear definition of the compass of inertia, proposed as rotation axis, and allows closed time loops, which as already observed, is the case of non-orientable manifolds. It was shown by Boeyens that these objections are raised if we drop the condition 5), by adding a point at infinity, which turns, as already discussed, spacetime to be non-orientable and identifiable as the projective space. In Gödel's theory, in terms of this simple alternative geometry, geodesic transplantation fixes points on the line element to occur along the double cover of a narrow Möbius strip, as we showed in figures 6 and 7 above. In those figures we showed that for the Möbius strip, a point, which moves along the double cover, close to one edge, rotates around the central line without intersecting, as depicted in fig. 24. This is what Gödel describes as rotation with respect to a compass of inertia. Any geodesic in projective Gödel space therefore rotates around an inertial compass in the same way. The constant separation between symmetry related points determines the number of turns before returning to the starting point. To define a non-orientable plane this number must be odd. All essential properties are recovered, without loss of generality, by assuming a single Möbius strip. On adding points at infinity to all coordinates of Minkowski space, *spacetime* transforms into the projective space, the conformal compactification of Minkowski space-time. More important than the precise topology of the modified space-time is that conversion to projective space, with five homogeneous coordinates, leaves the gravitational potentials unchanged, while the extra degrees of freedom represent the electromagnetic

field [59]. The Gödel solution therefore remains valid. In the geocentric mode, the Universe appears to rotate, as observed in radio astronomy, which would imply that Mach's Principle is invalid, as well as the isotropy of the cosmos appears not to be the case. As observed by Boeyens, the time paradox is also resolved in projective space. The closed time loops now connect any point, such as P , through involution, to its antipode P' in conjugate space-time. P and P' are related by CPT symmetry and separated in time. Yet, this separation in time is related to another most crucial law, in fact, a topological protoform of Newton's Third Law. Indeed, consider a normal vector placed at the point P on the Möbius band, and we rotate it 360° to reach for P' , its conjugate lying in the other side, localwise; now the vector points in the opposite direction and certainly is equal to the original vector, but now points out from P' , which is the local other side point of P ; see figure 24, right. Yet, this identity does not require, in distinction to the Third Law, no such assumption on the identity of action and reaction, which are not connected by duality no more; this is also the case for the periodic table of elements which on inclusion of the stable nuclides appears to have the Klein Bottle topology: In one side we have what the dualistic approach would consider an atom of matter, connected under a 360° path on the Klein Bottle to an atom of antimatter. We remark that the Klein bottle, can be thought as a section of the projective plane of disk type, as we shall discuss further below, while we recall that we already showed that the Möbius strip is such a section (see figure 1).

9. The non-orientable topology of the complex plane and its extension to the Riemann sphere

With regards to hyperbolicity in Relativity, consider a pseudocircle on Minkowski space $x^2 - (ct)^2 = \pm r^2$; it consists of two hyperbolas drawn in figure 15 (right). By following the asymptotes of the hyperbolas as an involuted closed curve e.g. ($A \rightarrow B \rightarrow A$) and $C \rightarrow D \rightarrow C$, two Möbius strips occur as sections through the projective plane. This path is analogous to the one that lies outside of the graph of the tangent function depending we approach, say, $\pi/2$ from the left or the right, which requires to identify ∞ and $-\infty$. In terms of the complex plane, this transformation occurs as the opening of the Möbius strip into its double covering with the following identifications

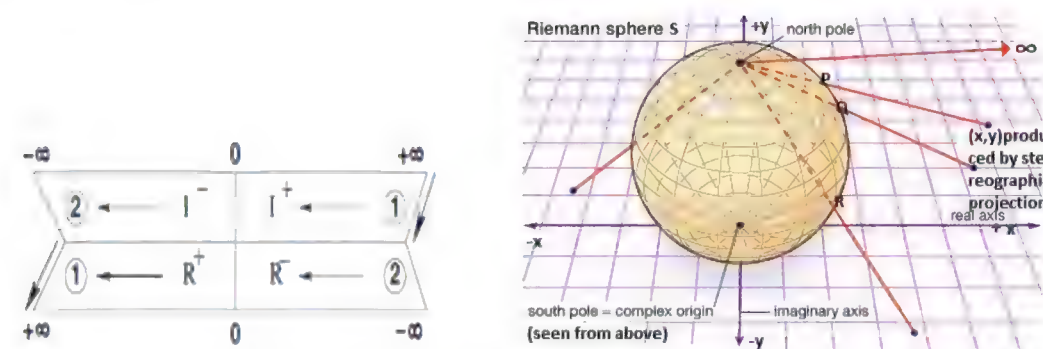
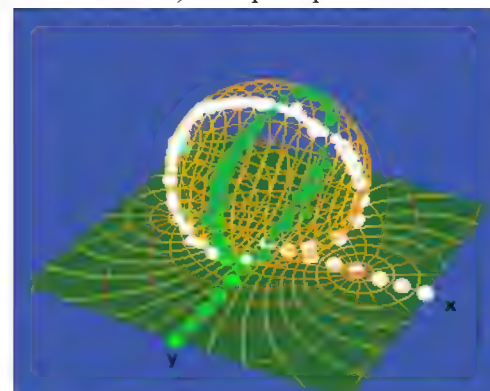


Figure 30: We fold the complex plane as the double covering of the topological identifications depicted above, proposed in [58]: . The folding of the Möbius strip from the extended (by the inclusion of ∞) complex plane as the double covering of the former; this non-orientable topology of



of the complex can be represented in the Riemann sphere in Figure 31, and is also the case of the visual topographic map on the brain's cortex.

Figure 31: We represent the non-orientable topology of the complex plane presented in fig. 30, as the 2:1 fundamental octave on the Riemann sphere, lifting the discontinuity of hyperbolic singularities.

In figure 30, we have depicted the unfolded double cover of a Möbius strip to model the relationship between real and imaginary numbers. Continuation of \mathbb{R}^+ beyond $+\infty$ at ① proceeds through ① at the positive imaginary semi-axis $I = +\infty$, with a twist as indicated by the arrows. The conversion from \mathbb{R} to I happens gradually in terra incognita near infinity. This characterization of the complex plane in terms of the Möbius strip invokes a discontinuity and a transformation of the real numbers on the imaginary numbers at infinity, which is filled by the action of the Time operator of the Klein Bottle Logic, which is the analytical continuation of the reals by a 90° rotation of the real axis to the imaginary one, as is the case of the analytical continuation on the time variable. Yet, in the Riemann sphere, S , given by the one-point compactification of the complex plane, the continuity is intrinsic to the development of the real axis on the imaginary axis. It is produced by moving (along a band on the meridian drawn as pointed white dots) from the South Pole/origin along the East meridian which corresponds to the positive real numbers until reaching $x=1, y=0$, and up to the North Pole/infinity; on reaching the North Pole we give a 180° turn (for which we have two choices, an East-ward or a West-ward pointing twist, i.e. a choice of chirality) to this strip, to continue with a different orientation (drawn as green dots) to the imaginary axis along the corresponding meridian, which now continues to the point $x=0, y=1$, corresponding to $\sqrt{-1}$ to further return to the South Pole/origin, thus completing the ① percourse. This is the first half-octave of the 2:1 harmonic of the Möbius strip and the Klein Bottle; the motion further rises following the meridian corresponding to the negative imaginary axis, up to the North-Pole/infinity, thus starting the percourse ②; on reaching the North Pole, we give a second 180° twist to the band, which returns to its original orientation/coloured surface, to follow now the West meridian corresponding to the negative imaginaries to complete ②. This yields the second-half octave and the completion of the 2:1 resonance. Since an even number of twists on the band have been produced, this indeed corresponds to the double covering of the Möbius strip, since the latter requires an odd number of turns. It is important to remark, that the 90° rotation on the complex plane on S that transforms $+1$ to $\sqrt{-1}$, corresponds to a 180° rotation from the South Pole to the North Pole followed by a motion (another 90°) to the point corresponding to $\sqrt{-1}$, and thus the 360° rotation on the complex plane corresponds to the 720° rotation and the motion South Pole-real axis-East-real axis-North-Pole-imaginary axis-South Pole-imaginary axis-North Pole-real axis-West-real axis-South Pole. We clearly see in this geometrical representation the 4π rotation of the double covering group of the Lorentz group, yet furthermore associated to the 2:1 resonance intrinsic to the Möbius strip and the Klein Bottle, as the transformation of the non-orientable topology of the complex plane, indicated by the figure, to the two sphere, whose local orientation is inverted twice. It is rather remarkable, that these characterizations have avoided recognition before. The change of orientation at ∞ along the meridians that allow to establish the continuity of the transformation between the real and the imaginary numbers is similar to that of the tidal forces on Earth's crust, in which the side pointing, say, to the Moon, with the most proximate moving point playing the role of ∞ , has an orientation opposite to its antipodals. Again, we have a 2:1 harmonic associated with this geophysical displacement of the crust, associated with the change of orientation of the gravitational attraction on the side of Earth exposed to the Moon, turning to repulsion in the antipodal.

10. The non-orientable topology and geometry of geophysics: pattern formation

Earth rotates precessing about its North and South poles axis, with a variable angle, presently about 23.5° ; the origin of this precessional motion can be found in the precessional motion of a spinning test particle (say, Earth) submitted to an exterior gravitational field with torsion [61]. This is the origin for the precession of the equinoxes and its signature on Earth is the Analemma; see fig. 12. In his *Philosophie Naturalis Principia Mathematica* Newton modelled Earth's dynamics in terms of its deformation as an homogeneous fluid body following the hydrostatic principle, and submitted to the Sun's and the Moon's gravitational attraction. In the 18th century, Clairaut raised the homogeneity condition to one of a liquid with fluctuating density along its radius. Later, other researchers

presumed the body's liquid to be an elastic and viscous substance. But the conditions of its hydrostatic equilibrium and inertial rotation of the planet have lingered up to recently reappear now as fundamentals of its dynamics as well as of the other celestial bodies.

It is currently understood that Newton's model is faulty. The study of oscillations of Earth's dynamics following earthquakes in 1960, lead to conclude that from the spectral decomposition of thousands of modes, only two modes appeared as the main ones: a spherical mode with direction of the radius, and a torsion mode normal to the radius (page 45, [76]). Of course, these two modes are well known in elasticity theory, yet, there is a most remarkable aspect that has been completely unconsidered in the geophysics literature, to the best knowledge of this author. The wave-fronts (or characteristics, in the theory of hyperbolic partial differential equations) of the equations of elasticity theory, in the linear regime, coincide with the wave-fronts of the Maxwell equations of electromagnetism [89], which themselves coincide with the wave-fronts of the Einstein's equations of gravitation, not only for a purely metrical connection [90], but one which also includes torsion, since in principle, they contribute in first-order of derivatives to the connection, while the characteristics involve second-order derivatives; in this setting, the gravitational field are given by harmonic equation. As we discussed in the Introduction, they are the fundamental signals/dislocations for the constitution of the physical world, yet they are of a self-referential nature.

These rotational elastic geophysical modes empirically verified are most visible on several places, and also appear as cases of the so-called unconformities of geology, in which newer stratae of depositions have surfaced ontop of older stratae, showing a circular time operator. These modes are very pervasive on Earth, visible through the naked eye (with the help of Google Earth) accounting for a design, and in several instances appear superposed to vortical motions on the atmosphere [75]. They appear to be related to harmonics of the fundamental 2:1 resonance, the very essence of the dynamics of the Möbius strip and the Klein Bottle. Their discoverer, Harold Overton, a retired engineering academic, calls it the Binary Theorem and relates them to the Croll-Milankovic cycles of the many-body problem posed by the Sun, Moon, Earth, and the other planets, the so-called *orbital forcing*, of crucial importance to the geophysical cycles [52,77,78,81].

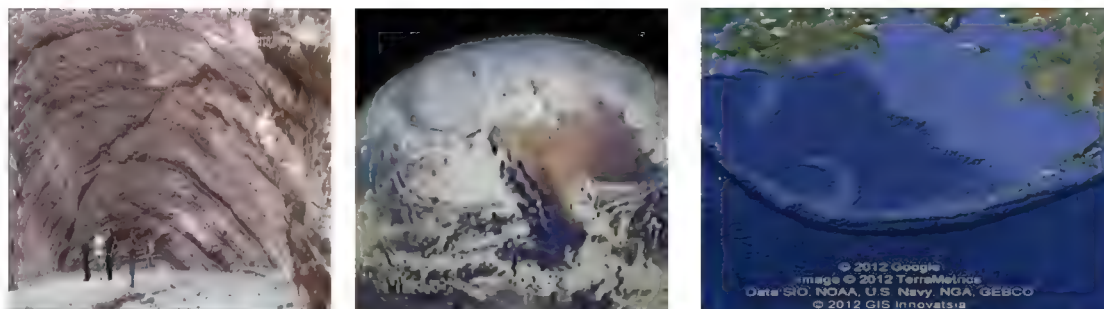


Figure 32. Left: Rotational crustal motions, Cafayate, Salta, Argentina. In [76] rotational modes are claimed to result from radial density inhomogeneities. Notice the dislocation from the ground climbing towards the right, and thus the lack of a well defined tangential direction of the tidal wavefront. Centre: Vortex over the Aleutians. Right: Partial view of the Aleutians Islands, a vortical crustal configuration, one of many in all scales (courtesy of Google Earth), indicating thus the existence of a time transformation of astronomical cycles forming Earth's crustal shapes, related to the 2:1 harmonic, and daily-scale climatological cycles, in which the ratio of the viscosity of the crust and that of the atmosphere play a crucial role.

A proposal is that the Earth is a self-gravitating body. Its matter moves in its own force field which is generated by mass particle interaction. The mass density distribution, rotation and oscillation of the bodies' shells result from the inner force field. And the orbital motion of the planet is controlled by interaction of the outer force fields of the planet and the Sun in accordance with Newton's theory. We remark two aspects of this proposal: firstly, it is about pointlike interactions of material constituents –

although it is recognized that the concept of point particles is untenable for it produces divergences in the mathematical computations so it is surmounted by considering charge densities (see page 52, [76].) and volumetric forces. Secondly, these volumetric forces cannot be reduced to applications to points but produce interior and exterior forces, so it is a dualistic approach, to address the formidable problem of the generation of Earth's shape, meaning by this the non-spherical shape, not the morphogenesis of crustal's shapes, which is left unconsidered. In this theory, the gravitational "anomalies" measured by satellites follow from the torsion oscillatory modes produced by the radial mass density distribution. as in fig. 32; see section 6.2 [76]. Other studies have found that the deviation of the hydrostatic gravitational potential due to the inhomogeneous density is an harmonic function (see chap. X, [53]); we recall that Fock showed that the equations for gravitation were given by harmonic fields [111]. Following these studies of modelisation of the interior of Earth, further extended to the crust modelled as a viscous fluid with displacement vector \mathbf{d} satisfying the Navier-Stokes equations (which is fundamental example of torsion geometry with torsion vector given by the velocity of the fluid [62]), we find that the displacement vector field is biharmonic and its gradient and divergence are harmonic vector fields. This shows a deep relation between elasticity theory and potential theory (page 499, [53]) further extend to Maxwell equations and the Navier-Stokes equations and turbulence [79,80]; it further shows that the torsion geometry of elasticity, potential fluid-dynamics, and the eikonal equations is of relevance to geophysics. For a thorough discussion of harmonic vector fields and the gravitational field we refer to chap. X [53].

In this model, we can think of the crust's morphology, as a frozen (in the sense of kinetic energy having transformed to heat) to reach a temporary stationary distribution of this fluid displacement, which is associated to harmonic vector fields. Yet, the latter are keenly associated to minimal surfaces in classical differential geometry, which thus would be an embodiment for the morphology of Earth's crust.

Kiehn has developed a topological theory of physics which unfolds with a common foundational setting than the present one in which the electromagnetic signals with wavefront discontinuities play a fundamental role. The evolution of physical systems is related to these singularities [95], one of which is the photon [19,136]; furthermore the relation between minimal surfaces, electromagnetic signals, the so-called topological quantization program of physics (and its relations with quantum mechanics as a theory of ensembles [138]), spinors –even macroscopic-, irreversible thermodynamical processes and turbulence, and a chiral vacuum indistinguishable from the Lorentz vacuum except from the impedance of free space were discussed [136,137]. (It will turn out that one particular application of this setting is crustal morphogenesis, to be dealt with below). Kiehn discussed in this setting the relation between a solution of Maxwell's equations of electromagnetism for a complex electromagnetic field, \mathbf{M} , due to Bateman [93] and minimal surfaces in 4D [108], i.e. mappings on 4D Euclidean space whose position vector, \mathbf{X} , is harmonic. These vector fields can be represented as \mathbb{R}^4 -valued maps F defined on \mathbb{R}^4 , given by the expression:

$$F(x,y,z,t) = (a(x,y,z,t), b(x,y,z,t), v(a(x,y,z,t) + ib(x,y,z,t))), \quad (7)$$

where $v = \phi(a+ib) + i\psi(a+ib)$ is an arbitrary complex analytic function of $a+ib$. Bateman proposed a complex 3-dimensional vector, $\mathbf{M} = \mathbf{B} \pm i\mathbf{E}$, with complex electric and magnetic field \mathbf{E} , \mathbf{B} could be used to express both the Maxwell Faraday and the Maxwell Ampere equations for the Lorentz vacuum as one combined set of complex vector equations. Consider the map be defined by

$$(a,b) \rightarrow (X^1(F(a+ib)), X^2(F(a+ib)), X^3(F(a+ib)), X^4(F(a+ib))), \quad (8)$$

with F analytic, and X^k ($k=1,2,3,4$) harmonic. Define $\mathbf{M}^k = \partial X^k / \partial a - i \partial X^k / \partial b$. If \mathbf{M} is isotropic, i.e. $\mathbf{M}^2 = 0$, and their complex square is not zero on the domain of definition of \mathbf{M} , then \mathbf{M} is defined in

terms of a minimal surface which is regular (without self intersections or pinch points). Being \mathbf{M} complex isotropic, it is a spinor [94,103]. If \mathbf{E} and \mathbf{B} are real, this is the case except at points where \mathbf{E} and \mathbf{B} are identically zero. If both \mathbf{E} and \mathbf{B} are complex, then the associated minimal surfaces will have 3D images that are not always regular. In general, two dimensional non-regular surfaces may have singularities consisting of curves of double points created by intersections of two local surface patches, or of triple points consisting of intersections of three local surface patches, or of curves of double points which terminate on "Pinch" points within the interior of the surface. These three types of self intersection singularities are the only three stable singularities in the sense of Whitney. Recall that Whitney proved that any N manifold can be embedded in $2N+1$ euclidean space, and immersed in a $2N$ euclidean space. The induced surfaces may be orientable or non-orientable.

If the surface has no singularities, then the surface is said to be regular or embedded. The constraint of regularity implies that the surface normal vector never goes to zero over the surface, or the induced metric on the surface is always invertible. This implies that there are always two linearly independent directions on a regular domain of the surface. If the lines of self intersection are divergence free on the domain (meaning that they stop or start only on boundary points, or are closed upon themselves, then the surface is said to be immersed in 3-Dimensions; the latter is the case of the Klein Bottle where the tangential discontinuity to the surface arises from the self-penetration and can still be embedded in 4D losing its singularities. The points where the divergence of the lines of intersection is not zero are defined as Pinch points. Such surfaces cannot be immersed in 3-D. The Pinch points are signatures of the fact the surface resides in 4-Dimensions (as an immersion), and cannot be immersed in 3-Dimensions. In this loss of regularity, which is very much the issue of the theory of hyperbolic partial differential equations and the constitution of the material world shapes, we find non-orientability as its signature.

A flow vector field may have domains where it is irrotational or solenoidal (we already saw that the displacement vector in the fluid crustal motion Navier-Stokes model is both), and these domains may be separated by a surface. If the surface of separation is a minimal surface, *then the flow on this surface is harmonic*. The minimal surface need not be regular, and may have lines of self-intersection. These lines of surface self-intersections (lines of singular double points) are not necessarily solenoidal. In fact, the Pinch points are points where the lines of self-intersection terminate not on themselves and not on a boundary, but in the surface interior. The Pinch points may be viewed as the "sources" of the divergence of the lines of self-intersection.

In terms of viscous fluids satisfying the Navier Stokes equations, if a velocity field \mathbf{V} (alternatively the displacement vector in the geophysical model already discussed) started out with both harmonic and non-harmonic components, then after the passage of time the flow should reduce (by viscous dissipation) to the harmonic components alone. Such is the theory of wakes. These wakes correspond in the case of non-regularity of the minimal surfaces to an evolution in which dissipation has occurred, reaching to a stable evolution. Indeed, for incompressible fluid dynamics, with $\text{div } \mathbf{V} = 0$, an initial (perhaps turbulent) fluid velocity distribution would decay by shear viscosity processes, such as those encoded by the Navier-Stokes term, $\nu \nabla^2 \mathbf{V} = 0$. However, any components of the initial velocity field that are harmonic, $\nabla^2 \mathbf{V} = 0$, will not decay, and it was argued that these components of the initial velocity distribution are those that form a residue or wake. The residue velocity fields admit a Hamiltonian representation, and therefore have a persistent existence: the initial domain with dimension four has been reduced by the evolution towards harmonicity to dimension two, the system organizes as a conservative classical mechanics (symplectic) system. Returning to consider the geophysical fluid model, either by the celestial interactions as orbital forcing or by motions arising from the interior of Earth, these stable patterns may suffer dislocations which provoke earthquakes, tsunamis, and appears that climatological vortices are generated superposed to them [75], and yet they retain their stability due to the dissipation of vortical motion in the form of heat, until a new temporal cycle starts another evolutionary decay to the harmonic configuration, and so on.

Therefore, the observable patterns associated to the gravitational field, which as we have seen can be interpreted as an electromagnetic field or a viscous fluid, or an elastic medium, correspond to

minimal surfaces in which their tangential discontinuities, observable features of hydrodynamic wakes which can be put in correspondence with those characteristic surfaces of tangential discontinuities upon which the solutions to the evolutionary equations of hydrodynamics are not unique. Only the robust minimal surface subset, associated with a harmonic vector field, will be persistent and of minimal dissipation.

We consider the equations for space curves in the plane, which are generated by a pair of differential equations of the form,

$$dx/ds = \sin(Q(s)), \quad dy/ds = \cos(Q(s)). \quad (9)$$

Since the rhs of this system is a unit vector, $\mathbf{t}(s)$ it is possible to identify it with the system of Frenet in the classical differential geometry of curves, with s being arc length, and $\mathbf{n}(s)$ the normal vector. The system of derived equation is

$$d\mathbf{t}(s)/ds = \kappa \mathbf{n}(s) = dQ/ds (\cos(Q(s)), -\sin(Q(s))), \text{ with } \kappa = dQ/ds. \quad (10)$$

A simple set of solutions can be readily identified: a simple sequence is to be recognized : $\kappa = s^{-1}$, $\kappa = s^0$, $\kappa = s$, the Logarithmic spiral, circle, and the Cornu-Fresnel spiral (relevant to diffraction patterns and the Analemma), respectively, all of them pervasive in the Earth's crust, as examination with Google Earth will demonstrate (see also [75]). Would we consider the characteristic shapes of intrinsic space curves for which the curvature is proportional to an arbitrary power of the arc length, $\kappa = g(s) = s^n$, for all positive and negative integers (or rational fractions), of special importance are $n=1$ and $n=2$,



Figure 33. Left: The Cornu spiral, is the half-year trace of the Analemma. Right: The Mushroom wake. The asymptotic involutes in the Cornu spiral as a dipole system are actually reached at the time of the equinoxes, when the time seems to stop with the day and night length being almost constant during a three days lapse after which the Sun distinctly arises above the horizon initiating a new cycle of life. Let us show some vivid examples of their crustal patterns:

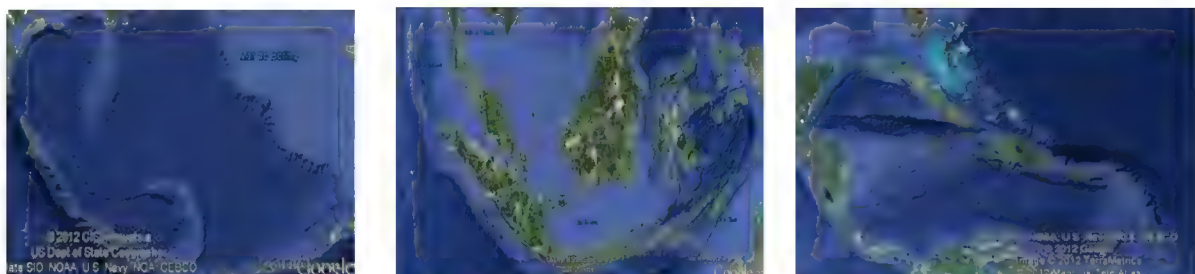


Figure 34: Left: Aleutian Archipelago. Centre: Andaman Sea, Indonesia and Philippines (notice the subeddies in major eddies). Right: Caribbean Sea. (the figures are deformed for lack of space; certainly their shape is kept).

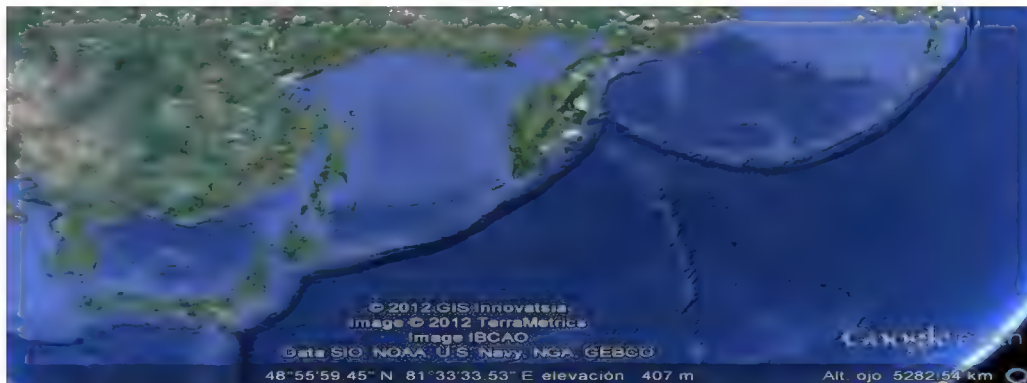


Figure 35: Left to right, the Japanese and Sakhalin Archipelagos and the Kamchatka Peninsula, extending Northeastwise to the Aleutians, along a common dislocation curve that extends to connect to the East Pacific to run west of the whole American continent and southwise to the Northeast of Australia. The tangential discontinuities are visible.

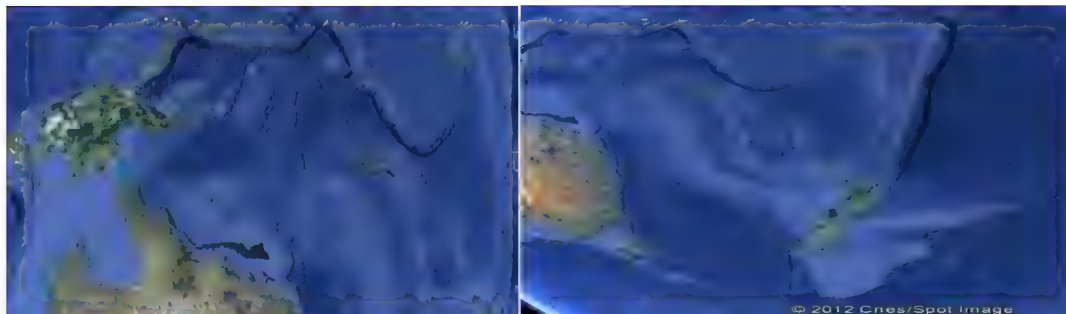


Figure 36: Northeast and Southeast Australian Sea; the tangential discontinuities are notorious.

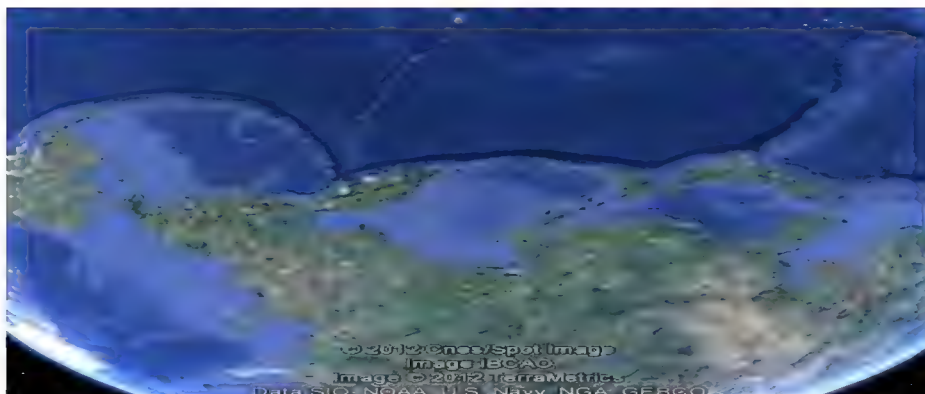


Figure 37: We follow instead the wave front of Figure 35 in direction of the North to return to the North of Asia, or we turn our attention Northwise to next figure:

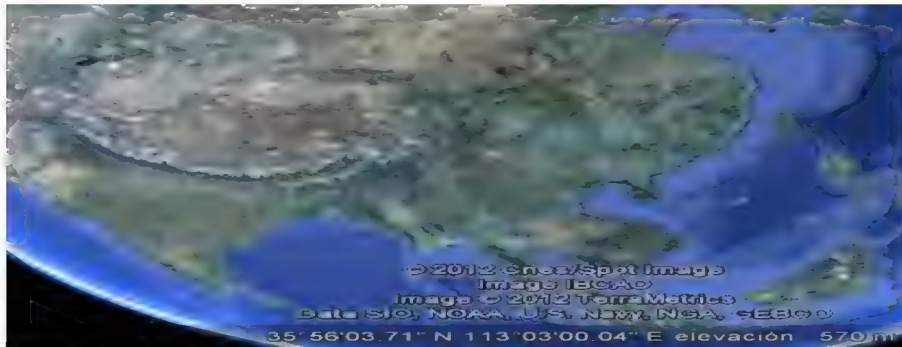


Figure 38: The Analemma formed by counter-rotating cells, on the left the Himalayas for its southern boundary, on the right China and Mongolia

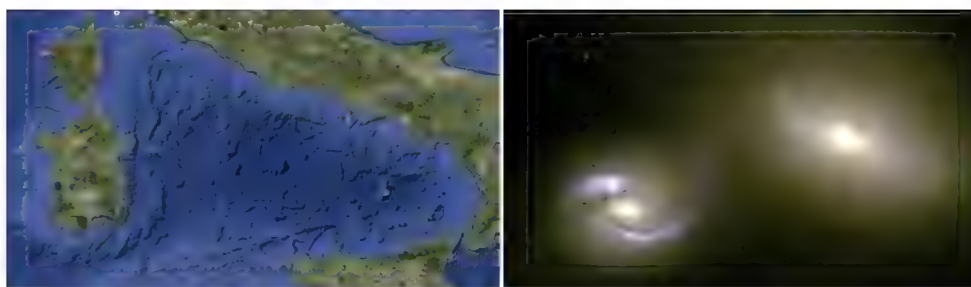


Figure 39: Left, the counter-rotating Analemma: Corsica and Sardegna, yet on a larger cell comprising the centre and south of Italy. Right: Galaxies NGC 1409 & NGC 1410 (courtesy of NASA).

Or we can change the scale to regard

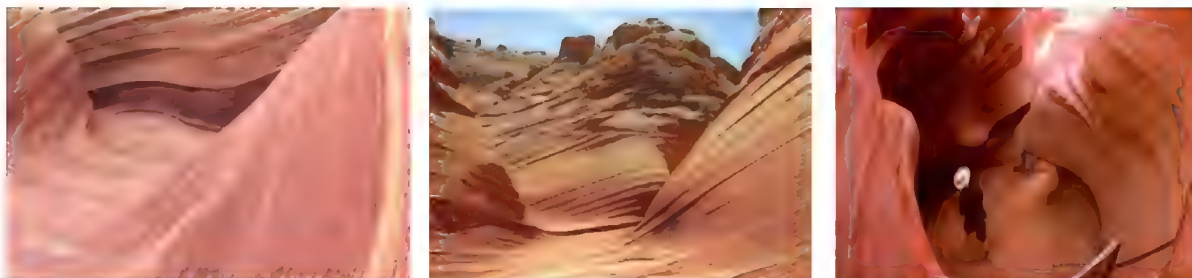


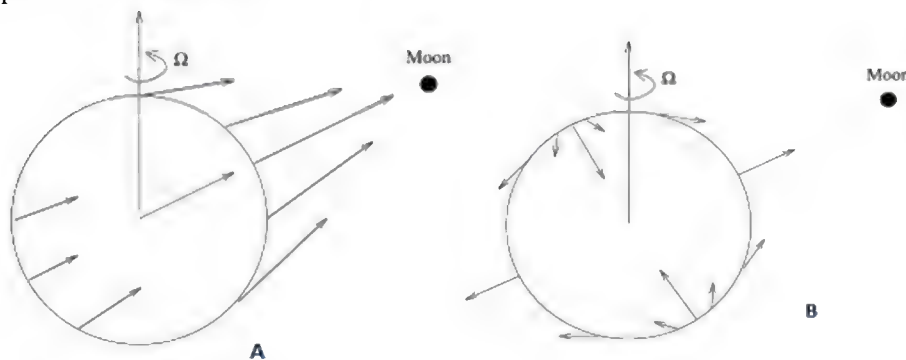
Figure 40. Left and Centre, a tidal wave: The Wave, Paria Plateau Canyon, Vermillion Cliffs National Park, and Antelope Canyon, USA, families of Croll-Milankovitch rotational cells.

The present conception as applied for crustal formation has also a theoretical link with what is claimed to be a new paradigm for systemics that stresses the fundamental character of torsion and vortical motions and structures (even in economics). In distinction of the linear attraction (with spherical symmetry) on points promoted to densities, the gravitational force is an eddy motion produced by the gradient of the gravitational potential with constant masses replaced by inhomogeneous densities [55,106,107]; more of this below.

The gravitational force on Earth produced by Sun and Moon, causes its orbital motion; in fact, it is this motion the largest effect of that force, which it further produces the deformations on oceans and Earth's crustal surface, Earth's tides. The main gravitational factor is the Moon and secondarily the Sun which accounts for about half the lunar attraction due to the distances; the other celestial bodies play a lesser yet still observable role; yet, other very important factor playing in the crustal morphogenesis and dynamics, is the amount of heating provided by the Sun, according to the Croll-

Milankovitch harmonic cycles [77,78]. While on the sea the gravitational attraction produces tides that may be as large as 10 m, the crust may move about 10-50 cm, and 10-20 cm in coastal areas.

The lunar-solar gravitational force can be decomposed into a part that is constant over Earth and causes the orbital motion, and still a variable on time and space force given by the remainder of this decomposition of the gravitational force. Thus we have the total force directed to Moon, the orbital force being an average of these forces, which in an approximation equals to the force on the origin. Would we subtract this net force then we get another form of the gravitational force described in figure 41. Now the tidal force is towards and away of Moon and is radially inwards on the other sides. This inversion of the direction can be thought as being a change of orientation of the tidal force along Möbius strips covering Earth's crust along the meridians. This pattern remains fixed with respect to Moon, and Earth rotates around Sun. This causes the tidal force at each point and surface on Earth to be changing on time. So the mass distribution on Earth changes with time. Say, for the tidal force produced by Moon placed nearer to the Equator, every point in and on Earth, would travel over two bulges and two depressions of the tidal force and thus the period of this tide is $\Omega/2$, two cycles per day, i.e. 12 hours. Consider instead that the Moon is placed forming 45° degrees with the equator; thus a point starting in a bulge at A moves after 6 hours it has moved to a inward depression at A' and returns to the original configuration in 24 hours; this is the case for any point in and on Earth. The period is now 1 cycle per day, the diurnal tides. If instead we consider the Moon to be over the North Pole, then any point on and in Earth would remain in ∞ . Since rather than being fixed Moon is orbiting Earth with a period of 24 hours, we have a combination of the three cycles, diurnal, semidiurnal and infinite periods. Thus the tidal force can be represented as a Möbius strip inside a toroidal configuration, the latter representing the diurnal tide, and thus we have the 2:1 fundamental harmonic. We also have an infinite number of strips wrapped on the original one, which account for the unending period. It is quite remarkable that this is identical to the topology of the mammal heart, as characterized by the Ventricular Myocardial Band proposed by Torrent-Guasp, the similarity stopping short of being an identity on the grounds that the superposition of bands to the original 180° twist that produces the heart, is not infinite. But then we recall that neither is the case for Earth on which the infinite cycle is asymptotic not actual, by the very nature of the North Pole being a point, i.e. unextended, and thus virtual, while what is physically manifest, say in the Riemann sphere, is the change of orientability in the neighbourhood of the North Pole., depicted in fig. 31. Thus we find that the gravitational (dominant) tidal forces, on the ocean and crust can be thought as distributed along Möbius strips (which we shall later propose to be better described by the Cross-cap) and still we find approximately the 2:1 harmonic characteristic of the Möbius strip and the Klein Bottle. They are visible on Earth's crust (and appear to have atmospheric associates) with the form of Analemmas of all sizes [75] established in what we can identify as the Shepard harmonics $f_n = f_0 2^n$, where f_0 is a fundamental frequency, that appear in music perception and its Klein Bottle representation to be presented further below.



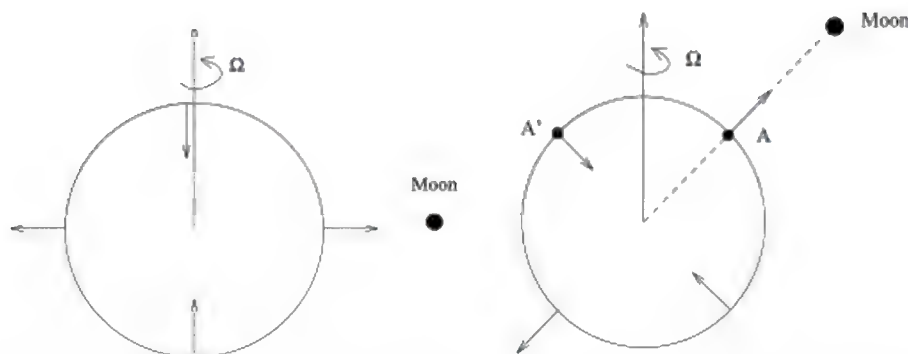


Figure 41 (reproduced from [59]). Fig. A represents the attraction of the Moon (or the Sun) on Earth's crust; Ω represents the earth's rotation, which we remark that it is variable, which is of crucial importance to crustal configurations due to tides, storms, hurricanes, earthquakes [55,105,106], and to climatological Croll-Milankovic cycles [64], of crucial importance to paleogeology and paleobiology, and to pattern developments and growth of molluscs, as first observed by Leonardo da Vinci [65], . Note that the total force at every point is directed toward the moon, and that the arrows closest to the moon are the largest (the difference in arrow length is greatly exaggerated in the figure). Fig. B represents the attraction-repulsive tidal forces when the constant attraction forces are subtracted from the former representation. We remark the change of orientation of the tidal force on the crust due to the depressions countering the bulges yet unified along meridians in which now the protoform of Newton's Third Law is the case. Note also that rather than an homogeneous force and thus gravitational acceleration, we have inhomogeneous acceleration is the case. This is the basis for the geoid, the non-spherical non-homogeneous gravitational equipotential surface; variations in the height of the geoidal surface are related to density anomalous distributions within the Earth. This validates Newton's Second Law for a non-constant mass and acceleration and that gravitation, rather than being a linear force, since the gradient for the gravitational potential with densities replacing point masses, generates a stirring motion and eddies, as discussed in [55,105,106] coinciding with the volumetric account of [76]. These stirring motion and the associated eddies are further related to the 2:1 fundamental and higher harmonics, and show up on Earth's crust and on the atmosphere as Analemmas of different scales corresponding to these harmonics.

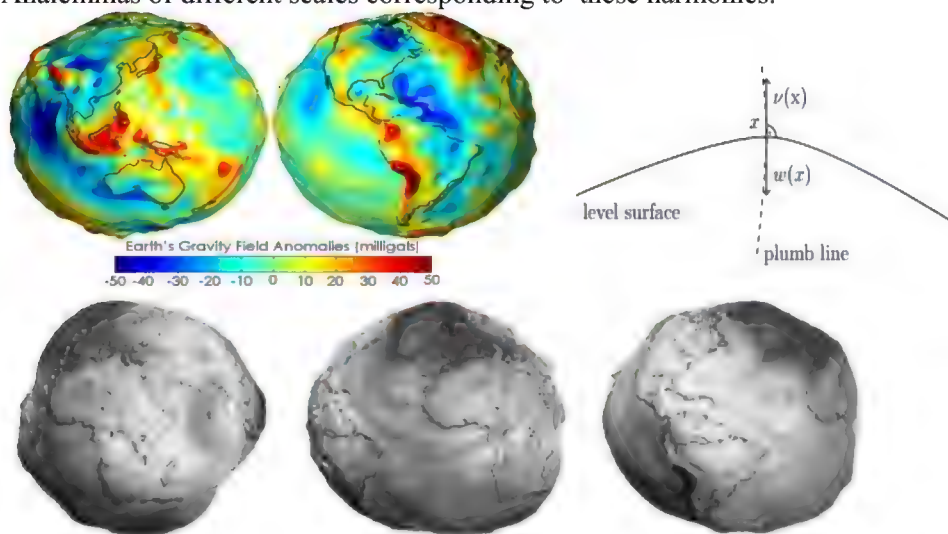


Figure 42. The geoid (courtesy of NASA). This shows that the gravitational potential is better described by its actions on extended areas, rather than virtual unextended points, the latter being the case of current geodesy. Alternatively, the gravitational constant becomes local (non-constant). Gravity, as observed on the Earth's surface, is the combined effect of the gravitational mass attraction

and the centrifugal force due to the Earth's rotation. The force of gravity provides a directional structure to the space above the Earth's surface. It is tangential to the vertical plumb lines and perpendicular to all level surfaces.

Consider now Fig. 43:

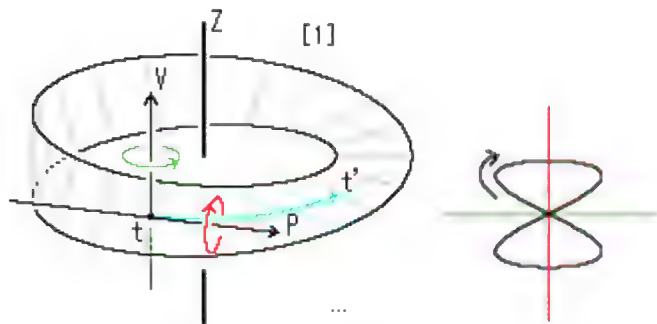


Figure 43: Tracing areal elements and motions on the Möbius strip. Right: the Analemma formed by tracing the motion of the surface elements on the 2:1 harmonic.

[1]: Z is the vertical axis that stands at the center of Möbius strip; it stands for the plumb line. The point t is moving to t' now under the rotational motion about Z of Earth. A small area at t somewhat turns around forward direction tP as shown the red arrow. Simultaneously the small area somewhat turns around tV that is in parallel with the axis Z . We can obtain the real twist of Möbius strip if we gather these two kind of twists. What we say "real" is the case we see Möbius strip from outside. Yet, Z if considered to be the vertical axis of the torus from which we cut the Möbius strip, we can further tilt it, as is the case of the precession of Earth's wobble around the North-South axis whose variable tilt is currently about 23.5° . As for the vector P we can think of it as a vector pointing in the direction of Earth's daily rotation or still the circumsolar rotation. Of course, we recall that it is the center of Earth that performs an elliptic orbit, yet the rotation of any element of area rather traces a 2-torus from which we can extract an embedded Möbius strip, as is the case above. Thus, rather than considering single points on Möbius strip for the origin of 2:1 harmonic it is elements of areas, i.e. densities, which are subjected to the gravitational pull of Sun and Moon, thus producing the crustal, ocean and atmospheric tides. This is consistent with the notion that the gravitational potential's action is not on ideal mathematical points but on actual physical extended areas, densities, so that the Newtonian expression for the gravitational force between two masses is substituted by two densities, and thus the gravitational potential given by the gradient creates a stirring vortical motion, rather than a linear motion as usually conceived. Whatever the angle of tilt of Z is with regards to the orbital rotation of area elements, the view from the left of these 2:1 resonances is a figure 8, i.e. the Analemma, yet not necessarily solely corresponding to a 2:1 harmonic produced by an annual rotation, but rather of cyclic phenomenae on Earth's crust and in growth of natural patterns [56,57]. This captures the two main oscillatory modes previously discussed that were found from measurements, yet combined in the two superposed motions described in Fig. 41. Let us give now a possible alternative topological description, that still captures the non-orientability, and still the fact that the gravitational field may, in this theory, be related to a projective spacetime.

11. The crosscap and non-orientability in geophysics

Cross-cap or projective plane can be divided into Möbius strip and a disk. It will be easier for us to observe relations among the representations of the non-orientable projective plane, Cross-cap and the Möbius strip.

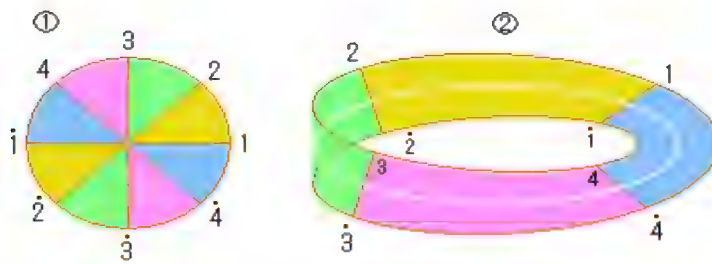


Figure 44: The cross-cap on the left and its representation on the Möbius strip on the right. Would we identify the antipodal points, we get the Klein Bottle, as shown in Fig. 10C.

①: We divide the disk into eight sectors equally by 4 diameters and color it as shown. The center, however, can not be divided by colouring. The end points of every diameter are numbered.

②: Disk ① became Möbius strip. The center of the disk is a single point, and now it is the center line of Möbius strip. Every coloured area corresponds to that of the disk except the center of disk. One to one correspondence stands up in the coloured area.

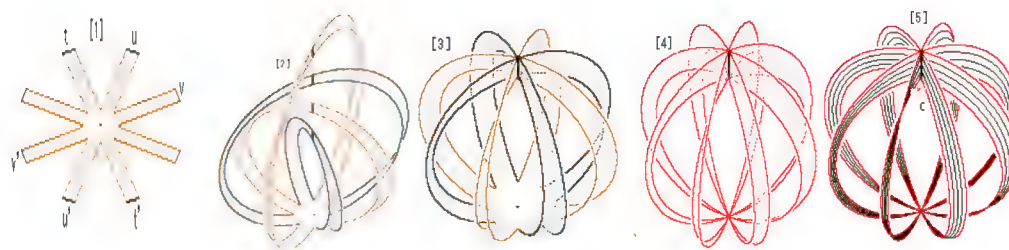


Figure 45 (courtesy of T. Ito): Surgery on the cross-cap.

[1]: Disk-type projective plane is a circle, which we cut the disk into rectangular strips arranged around the octagon that surrounds the center of disk, which is excluded. [2]: We join every pair of antipodal edges after twist. The twisting directions are made evenly. The black bridge on each thin strip is the joint line. Thin strips look like octopus legs. They are Möbius strips. Colours of the legs are changed alternately for easy observation. [3]: We make black joint lines free to pass through, and we let Möbius strips intersect each other as shown. [4]: The octagon is shrunk to a single point. It is what the identification of antipodal points of [1] is done. The edges are coloured red in order to show that they are diameters of Disk-type projective plane. [5]: We split Möbius strips thinly. We assume that they become the diameters. [6]: It looks like an apple, it is made of the diameters. The joints of Möbius strips is the segment c now. The Cross-cap representation of the disk-type projective was seen to contain along the meridians of the sphere produced by antipodal identifications being glued, make of the projective plane a sphere whose meridians are Möbius strip, which we assume arbitrarily thin. Yet, on the case of the gravitational stirring force that produces the change of orientability associated to the tides, this is no longer the case. Thus, we are lead to propose the notion that they correspond to the natural topology of Earth, with regards to tidal forces, particularly of its crust, which are crucial to its geophysics. The zonal manifestations of this local changes of orientability is the appearance of Analemmas on all scales on Earth's crust and the atmosphere [75].

12. Blow-ups, divergences and the non-orientability of non-linear evolutions

The existence of time loops as reentrances of systems alike to the Gödel solution is by no means something exclusive of the hyperbolic geometry of Special or General Relativity, since it is further related to the existence of divergences in arbitrary non-linear systems. These discontinuities that involve the numerical notion of unbounded growth, is associated with the notion of blow-up, and they reflect transitional changes (the blow-ups) of systems described by, say partial or ordinary *non-linear*

evolution equations. In the science of well posed linear systems, the assumption of initial conditions is the backbone for determinism and prediction of future states from the fixed initial state; yet linear systems do not have the ability to predict forthcoming transitional changes and the establishment of new systemic regimes posterior to them, which are the province of non-linear equations. In terms of non-evolution equations, blow-ups embody the destruction of old structures and their replacement by new ones. Prigogine already claimed that Einstein's theory of general relativity with its non-linear partial differential equations, lacked the historicity of evolutionary processes for the reason that time in the contemporary conception going back in science to Newton, is a mere parameter[105]. So the appearance of new regimes is not the exclusive province of non-linearity, but also of a conception of time which supersedes the mechanical notion of a mere additional variable to be compounded with space. Of course, in the theory of non-linear equations, the assumptions of initial and boundary conditions limits the ability of the equations themselves to account for interesting physics, climatology, mathematics, etc. [55,105,106].

Thus the concept of blow-up introduced by Yi Lin and Soucheng OuYang, reflects a very general phenomenology. For a given mathematical model, that we assume to describe a system's evolution, which we characterize by $u = u(t; t_0, u_0)$, with t the time variable, that satisfies $\lim_{t \rightarrow t_0} |u(t; t_0, u_0)| = +\infty$, and also, when $t \rightarrow t_0$, the system in question goes through a transitional change, then the solution $u = u(t; t_0, u_0)$ is called a blow-up. For non-linear systems in independent time and space variables, the blow-up is defined similarly. The question of this transition is what the meaning of ∞ is, rather than the mathematical indeterminacy that the real numbers unboundedness poses. The striking identity of this with the mapping of the double covering torus topology of the complex plane of its non-orientable topology as characterized by figures 30 and 31 is that the blowup rather than a divergence lifted by the one-point compactification of the complex plane represented in the Riemann sphere, it expresses the local change of orientability; already Yi Lin and OuYang claimed that the blow-up was equivalent to the passage of the system through the North Pole $= \infty$, showing a reversal of $+\infty$ to $-\infty$, yet they did not identify the relation of this with the topological issue of change of orientability, which we have found to be pervasive to science at large. Following these works, it is apparent that the issue of non-orientability is also crucial to the so called chaotic systems, as already the 2:1 resonance immersed in the 2-torus shows to be the case. Remarkably, ∞ corresponds to the Möbius twist superposition cognitive state in the Matrix Logic representation of the Klein Bottle Logic, since it is both positive and negative, and nor positive nor negative, so that the change of orientability on N is the precise manifestation of this superposition. Boolean logic true and false states are the projection by the Hadamard operator which in this logic is the matrix representation of the Klein Bottle, which by further application of this operator, we retrieve the superposition states [14]. We see that ∞ is not associated to Boolean logic, but to the Klein Bottle Logic. This is of crucial importance on regards to hyperbolic divergences that arise in non-linear systems, which have been noted that are indeed outside of the cognitive realm established by Aristotelian-Boolean logic, as claimed in [55,105,106] Thus, rather than problems of indeterminacy of non-linear systems, blowups of non-linear systems *undergo a change of orientability as the topological signature of the change of regime with the appearance of new structures and processes*. This amounts to the manifestation of chirality following the explanation of figure 31 on the significance of transversing through the North Pole from the real to the imaginary axis with two different modes: Would the blown-up system reenter through the origin with imaginary values (green dotted path), then the chirality that has manifested with the first twist at infinity is preserved. Instead, would the system reenter through the negative real values (on the white dotted path), then the previous chirality is flipped by the second twist at infinity, and thus we have a complex orientable plane.

This reveals a deep meaning of singularities which has been overlooked. Yet, non-linearities is the case of vortical motions, as transpires already in the fact that the velocity field of the non-linear Navier-Stokes equations of fluid-dynamics is the torsion field, whose differential is the vorticity, and still the magnetic field of the kinematic dynamo equations of magnetofluid-dynamics. In this case, the vortices appear as a gradient of the velocity field of the inhomogeneous shearing motion of

proximate sectors of the viscous fluid, and is also the case of magnetohydrodynamics [62], where the non-orientability of plasma flows as already manifested in solar coronal ejections.

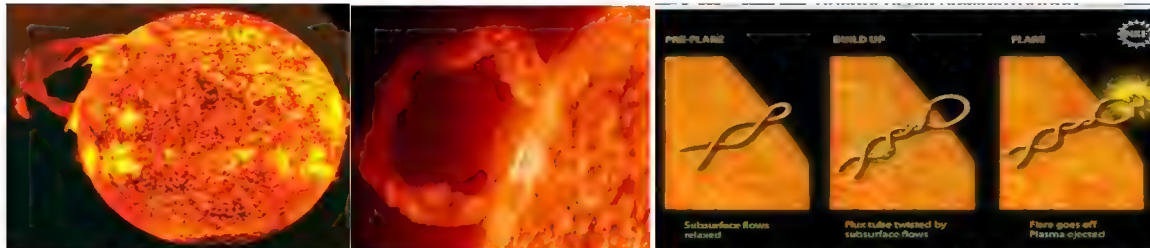


Figure 46: Lhs and centre: Solar coronal ejections; in the rhs the model by the National Oceanic and Atmospheric Administration, underdescribes the change of orientability that is manifested in the ejections (courtesy of NASA & NOAA, respectively)

In this conception, time is unseparable from material inhomogeneities as argued in the theory of blow-ups [55], yet this is so because the logic of the physical organization of matter and fields is the Klein Bottle logic of reentrances, and this extends to thinking and cognitive processes [9-16]. Eddy motions arise from the unevenness of space and materials [55], and the time cycles as occurs in Earth's crust and atmosphere, and the unevenness of time as argued by Kozyrev [67,68,70] (and empirically tested in very different situations, such as rotating tops [69], physiological cycles of plants, torsion balance anomalies and other phenomena) in which efficient transformations or kinetic energy to heat occurs, with the consumption of kinetic energies establishing an equilibrium between them. But even more crucial, this shows that the notion of forces as exterior to systems, in the case of unevenness, is the physical manifestation of inhomogeneities. Yet, more fundamental to them, is the protoform of Newton's second law, embodied in the non-orientability of the Möbius band and the Klein Bottle, or still, the projective plane. Thus, with respect to Newton's third law, $F=ma$, as already the geoid shows, the acceleration a is no longer constant (but actually an average if considered as acting on an ideal point), nor is the mass m , and we rather have than a force applied to a non-extended virtual point of space or spacetime, a force that arises from the uneven distribution of a material or space. We can express this by saying that Newton's third law is contextual, and its absolute character resides precisely in that the physical constants are not such objective values but contextual variables which are, in usual practice, averaged out, losing thus the irregularities that rest at the physics of eddy currents and motions. Thus, instead of the gravitational potential, say acting on unmaterial virtual points of Earth's crust as assumed in the more standard geophysics [54], we have the gravitational attraction acting on the densities of real non-ideal extended matter distributions. Otherwise, it would be impossible to understand how morphogenesis of Earth's crustal structures and processes occurs, which is the case as Analemmas at several scales, and is also the case of the atmosphere with the occurrence of vertical structures both crustal as well as atmospheric. Furthermore, as much as the Time operator of the Klein Bottle Logic which geometrically is the $\pi/2$ rotation of the complex plane (identified with the cognitive plane) from the real to the imaginary axis, which represented on the Riemann sphere is a $3\pi/2$ percourse can also be seen through its action on two cognitive states of the Klein bottle logic, which is tantamount to compute its algebraic *difference*, systemic time can be understood associated to the dynamic *difference* that is embodied in the *unevenness* of materials. Thus, it is possible to understand the systemic time cycles that permeate Nature, and in particular the Croll-Milankovic climatological and geophysical cycles that we have argued that they arise from the 2:1 resonance of the Möbius strip as Analemmas on the Earth's crust produced by tidal action, or in the atmosphere, as the time operator that is intrinsic to this fundamental change of orientability that produces this harmonic resonance. Thus, generically the blow-ups are spinning currents which only appear with time, in which non-orientability supersedes the discontinuity proper of the divergences. With respect to the well-posedness of the initial conditions with their accessory uniqueness, existence and stability of the solutions of the non-linear evolution equations, they are not satisfied by nonlinear

problems, so that blow-ups will only be avoidable under special conditions which are very different to meet in practice. While the curvature of the Riemann sphere and in general of spacetime allows for the hyperbolic divergences to be mapped to continuous transformations as self-reentrances of the systems in question, as already the Klein Bottle Logic that sustains this self-reentrance it is the torsion fields and their topological operations that fold and unfold the reentrances of the systems as new regimes that in turn will fade away, numerically diverge and transmute to a new regime. The torsion fields are the vortical motions that embody the blow-ups.

With regards to the thermodynamics, it is very important to remark that on the case of non-linear systems described by the relations between entropy S , thermodynamic forces X_i and thermodynamical flows J_i , related by the non-linear equation $\frac{dS}{dt} = f(X_i J_i)$ that extend the linear case, and are the usual physical case, it follows that irreversible processes experience a blow-up, which signals the self-reentrance of the system following the increase of entropy with a finite time, reordering the system with the decrease of entropy [55]. It was argued that the cycles of re-entrance of systems is related to the Time Operator of the Matrix Logic formulation of the Klein Bottle logic, associated to the derivative of acceleration, control, neglected in physics but which shows up very pervasively even in quantum mechanics [55], and still the second-derivative of acceleration, self-control [15], which in the case of the Earth's crustal dynamics appears as the upthrust motion that inverts the geological layers, producing the older layers to rest upon the older ones [75]. This establishes a non-linear morphology of Earth's crust and atmosphere, as revealed in the figures we have presented above, and the notion of geological time. The latter was the subject of a much followed confrontation between Lord Kelvin and Charles Darwin, the latter claiming a uniform crust to avail his theory of evolution of species that required small uniform changes of the crust [107]. Leonardo claimed that biological patterns of molluscs on Earth were related to the orbital forcing due to the Sun and the Moon, which we know to determine the shapes of the crust, and paleobiology appears to support Leonardo's claim [56,57].

Therefore, rather than the universe expanding towards a heat death, we have a universe that reenters itself or to another universe through a Klein Bottle wormhole. The universe or the multiverse establishes an equilibrium through the cycle of destruction and recreation. This is the logophysics of all systems, a claim that begs further qualifications. We shall be brief, out of necessity.

In a lived world in which the anatomy-physiology of the cognizing-sensing-thinking-enacting subject, the physics and chemistry of systems, the logic and the fused perception and cognition of the subject, all co-conform the logophysics of self-reentrances of the Klein Bottle, we are still left to deal with the issue of complexity of interactions of monads conformed thus as well as to the constitution of each monad; this complexity transcends the reductive quantitative notion of algorithmic complexity, being of logophysical character. This requires the consideration of the HyperKlein Bottle in the framework of a hypersystemic theory of systems of self and mutual reentrances and closed/open containments, selfpenetrations and interpenetrations of Klein Bottles, which in the lived world are processed as a single integrated one [9,12]. Would we wish to recourse to a dualization (which as already mentioned, is a projection of the Klein Bottle Logic [9-12,14,15]), even in apparent contradiction with the principles of this lived world, claiming a specificity of a particular system due to complexity, it is this *hypersystemic integration and still relational specificity* that the HyperKlein Bottle embodies in a logophysics which the lived world through the body integrates *singularly*, and still *universally*, as a *single monad*, while preserving diversity, both perceptually and cognitively. In other words, cognitive complexity is maintained while integration is universal.

This monad whose reification waxes and wanes, is eternal; would a nihilification posture otherwise, it would stand in the previously mentioned projection of the Klein Bottle Logic to dualism, from which the superposition states (topological precursors of *Schrödinger's cats* states), by iterated self-referential action of the Klein Bottle in the Matrix Logic form of the Klein Bottle Logic, are recovered [14,15]. Thus, instead of the Nil/Void which quantum physics has long ago vanquished (yet physicists may associate with 0 [116]), the undifferentiated emptiness which logicians and cognition scientists appear to impose on the Plenum [124,125,127], it is this logophysical periodic oscillations of

the Plenum what constitutes its manifestations, eternal as self-referential self-reentrant action is. The reentrance occurs through logophysical time waves, imaginary logical values [127] first introduced by Spencer Brown in his *Laws of Form*, which are also basic to the genetic code [12]. When a system comes to manifestation in the physical world, then its eigenstates are real, and divergences, either by initial conditions or constraints will make it blow-up, and then the reentrance will occur as previously discussed. They are the signature of the eternal return.

13. Nonorientability, the human heart and vortical dynamics

The mainstream science model of the human heart claims that the heart (weighing about 300 grams) is a pump, which as a vessel its walls have a fixed width, and is capable of 'pumping' some eight thousand liters of blood per day at rest and much more during activity, without fatigue. In terms of mechanical work this represents the lifting of approximately 100 pounds one mile high [88]. In terms of capillary flow, the heart performs the task of 'forcing' the blood with a viscosity five times greater than that of water through millions of capillaries with diameters often smaller than the red blood cells themselves! Due to the complexity of the variables involved, it has been impossible to calculate the true peripheral resistance even of a single organ, let alone of the entire peripheral circulation. One of the key concepts implicit to this model is that blood is naturally inert and therefore must be forced to circulate. Remarkably, this conception was expounded by Galileo and Leonardo da Vinci, who in his Notebooks depicts the left ventricle wall as having uniform width, as

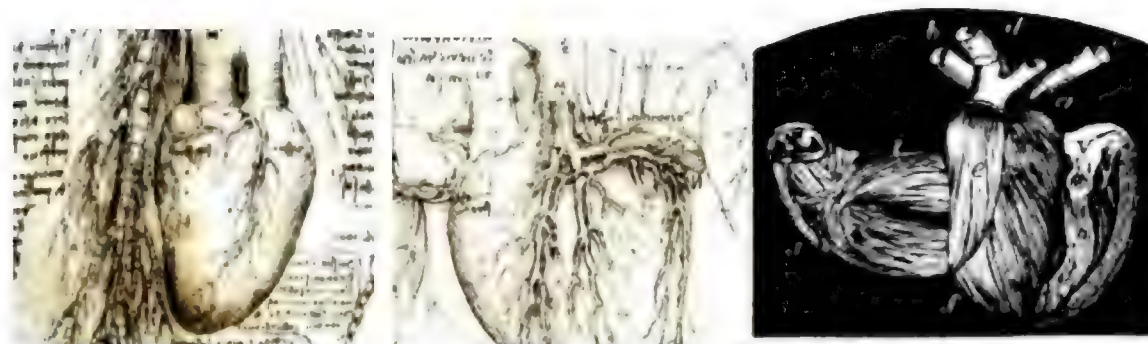


Figure 47. Left and Centre: by Leonardo da Vinci. Right: In stark contrast with Leonardo, John Bell Pettigrew showed in 1874, after mammal hearts were subjected to boiling and thus the fiber structures were revealed, the descending and ascending helical fiber structures surging from the apex on the bottom of a turkey's heart –anterior view–, revealing what Torrent-Guasp rediscovered almost one hundred years later. Pettigrew further gave a thorough discussion of the relation between the twisting motion of the fibers and the physiology of the heart [22,23], anticipating Torrent-Guasp [71,72]. He noticed the variable width of the fibres and their importance to its physiology. Currently, they are visible in vivo, through magnetic resonance imaging, and studied by the Biomechanics Group at Caltech [73]. We expect to find in a pressure chamber; quite the contrary, the left ventricle wall thickness varies by about 1800%, as was found by dissecting bovine hearts [71,72,88]. Furthermore, in the pumping conception the blood flow is passively shaped by the heart; this is the Cartesian concept of space as a container for objects, which was retaken by Newton and fully developed by Einstein in his theory of relativity, which excludes spin which requires torsion for its representation in a gauge theory of gravitation of the Poincaré group. To question the Cartesian model of the heart, it has been observed that the thin, flexible, cone shaped apex and suspension from the aorta suggest the accommodation of a twisting function especially, when taking into account the spiral orientation of the myocardial muscle layers. We cannot refrain on commenting the strangeness that this elicits with regards to Leonardo's description of the physiology of the heart, since he above all, was the master in the study of eddies in fluid-dynamics, designer and builder of canals and hydraulic systems, a unique pioneer in observing and representing Nature. In fairness, the limitations appear to rest on the

preparation of the dissected heart by boiling them, to be able to separate the layers of the tissue in dissolving thus the connective tissue.

The circulatory system can be seen as a transformation from a 2-torus to a Möbius strip at the root of the pulmonary artery and back to a 2-torus tube that becomes at the root of the aorta, as depicted in figure 48, left.

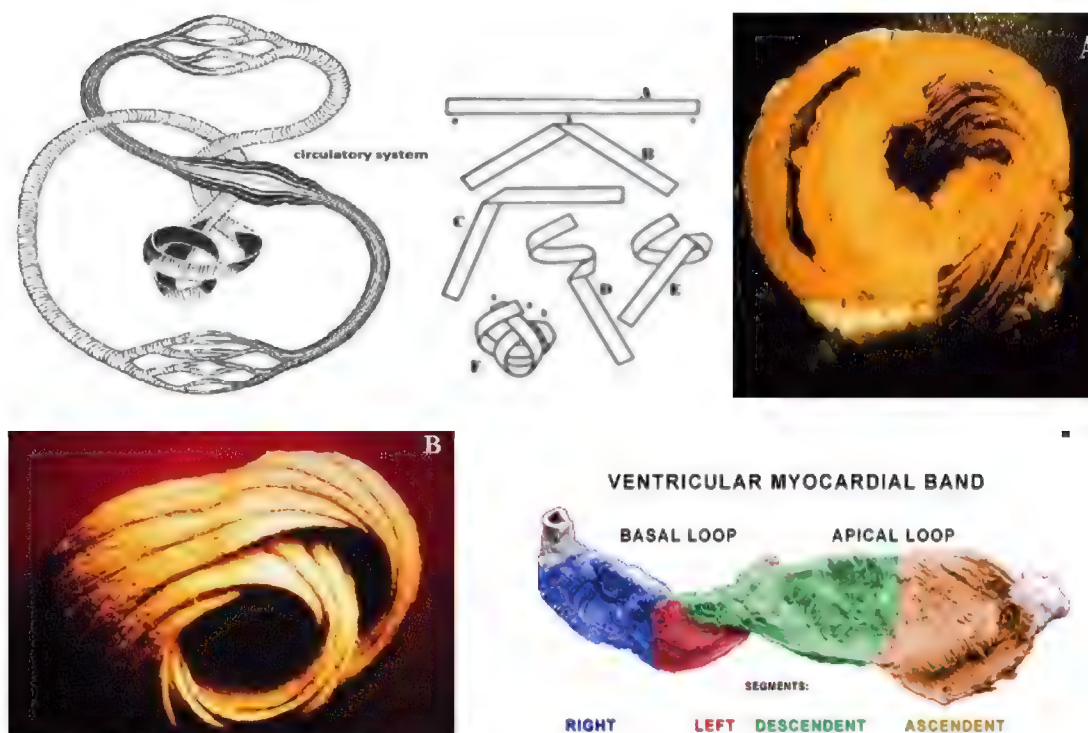


Figure 48: Left: 2-torus to Möbius strip to 2-torus transformation of the circulatory system. Center: We introduce the topological model on a paper strip of the Ventricular Myocardial Band by Torrent-Guasp. Start with a ribbon A (the pulmonary artery) on which we introduce a 180° turn, as in B, and C is the aorta, to form the apical and basal loops. Right and below: Unprecedented dissections by Torrent-Guasp A, transversal cut of the ventricles. B, posterior aspect of the ventricles seen from below. The left ventricular cavity is circular whereas the right one is semilunar. The right ventricular cavity cannot be considered as a true ventricle but, merely, as a fissure, a cleft opened in the thickness of the ventricular wall, alike to the separation of the Möbius strip from the 2-torus in figure 50 below. We note the iteration of the process covering the original single twist with a multilayered organ, the heart. The origin of the loop can be seen in the twist, in from the basal loop (in red) to the apical loop (in green) unfolding of the heart without any cuttings (photographs and drawings by Torrent-Guasp, courtesy of the Torrent-Guasp Foundation and its director, Dr Mladen Kocica).

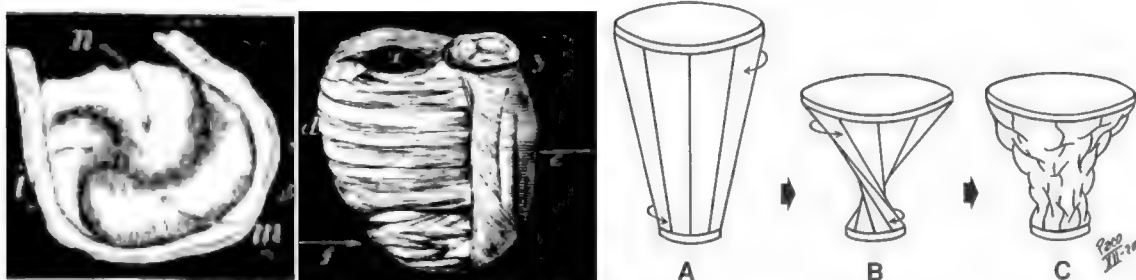


Figure 49: Left: "Tricuspid valve action seen in a human heart. The segments are folded and twisted into each other by a screwing motion (vide arrows m, i, n) occasioned by the spiral movements of the

ventricles and the spiral impulse communicated to the blood contained within the right and left ventricle" (Fig. 85, page 174, [22]). Centre: The fourth layer of helical fibres, showing the point in which the external fibers turn to the inside (Fig. 99, [22]). Pettigrew referring to the layers: "an arrangement so unusual and perplexing, that it has long been considered as forming a kind of Gordian knot in Anatomy. Of the complexity of the arrangement I need not speak further than to say that Vesalius, Albinus, Haller and De Blainville, all confessed their inability to unravel it", and consequently it was disregarded until Torrent-Guasp's resurfaced it and elaborated his model. In A,B, and C we show the lateral view of the torsion, lengthening and shrinking of the layers of the heart (by Torrent-Guasp, courtesy of the Torrent-Guasp Foundation).

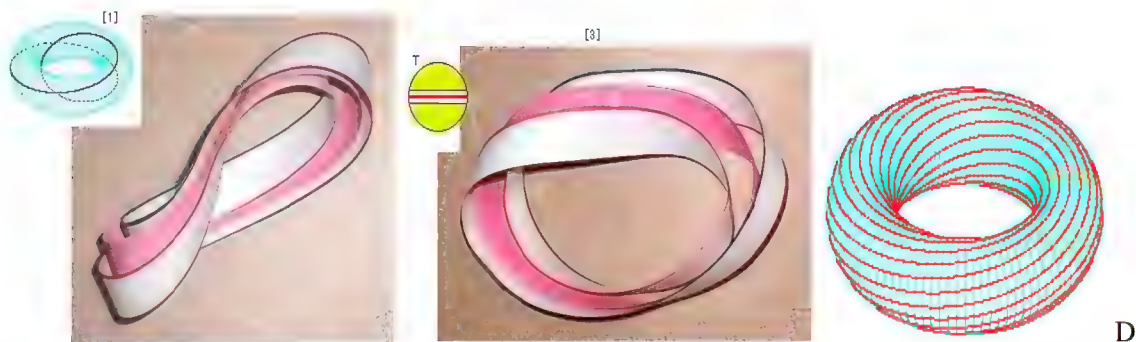


Figure 50: extracting the first layer of the heart in the multilayered heart that arises from the 2-torus cut of an iterated Möbius strip

Consider figure 50. [1]: We present the 2:1 resonance in the elastic 2-torus from which we have a Möbius strip in Figure 25. The black curved line is the track of cutting. [2]: The photo is a paper craft to show the situation at that time. The red belt is the Möbius strip and the white belt is the surface that contacted to it. They are loosened a little for easy observation. The white belt is two-sided. So we coloured edges brown and black separately. [3]: The picture is taken in different direction. T is the section of torus. Finally in the right we present a stack of n paths parallel to the 2:1 resonance which we can also cut to produce a multitier figure instead of the three-stack figure, yet it has one single twist so it is a long Möbius strip, yet with superposed red layers while the total angle of turns of the overlayed twisted red layers is $4\pi n$. This will be the topological model of the Ventricular Myocardial Band which indeed has a single 180° twist yet with superposed layers that make the heart's wall.

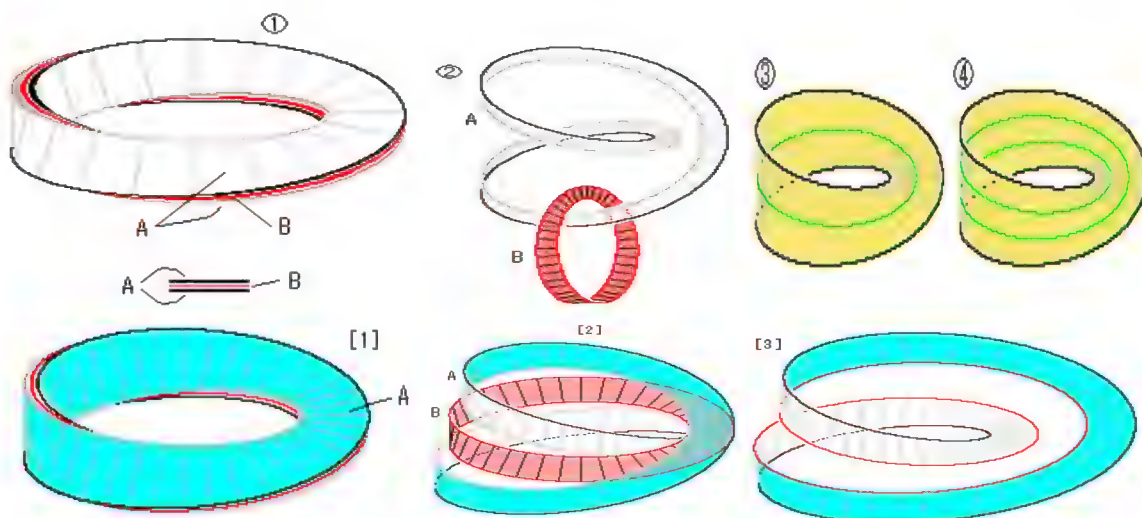


Figure 51. [1]: The belt painted sky blue is loop **A** in Fig.2. It is two-sided, and so we can distinguish its surfaces by color. [2]: The left parts of loop **A** are vertically pulled apart from Möbius strip **B** a little, alike the right ventricle in Figure 39. The white surface, rear side, of loop **A** appears now.[3]: Möbius strip **B** is omitted. The right parts of loop **A** are horizontally separated a little each other. The brown edge is coloured red now for contrast. The twist is 360° as long as we see this diagram. Roughly speaking, [3] is a coil of duplex winding. Iterating this by taking cuts along paths as in fig. 49D will form the Ventricular Myocardial Band. This topological model in which instead of paper we consider real elastic myocardial fibers may serve as a basis for a mathematical model of the heart in terms of the elasticity associated to the superposed harmonics of the iteration. Remarkably enough, the two-fold superposition of a Möbius strip, is a simple model of a continuous transformation of spin-one half into spin-2 unification states [95].

14. The non-orientable psychophysics of music cognition on, visual perception and pattern recognition.

Physicists and philosophers have studied visual perception and the geometry of visual space [97, 98]. The latter was a central issue to the Renaissance (Leonardo and Dürer its pioneers), which through projective geometry abandoned a trivial plane in which a fusion of object with subject operated, aiming to represent three dimensions on the plane. The identification on this plane, of object with subject, was the signature of the Middle Ages, evidenced in the lack of depth, the primal dimension of the lived world which phenomenologist Maurice Merleau Ponty associated with time [140]. Thus, the Renaissance introduced depth accompanied by a detachment of the subject to observe the world as an outsider, setting the stage and the scenery for Descartes. Lüneburg discovered that hyperbolic metrics (as in Einstein's Relativity) for which parallels converge, may describe the geometry of distant-object-vision instead of the near-vision Euclidean geometry, and that a psycho-metric function dependent on the observer is crucial to visual perception. Thus, a plural visual representation geometry (there is a third zone with a distinct geometry) was proposed. Thus the Principle of Non-Contradiction, the backbone of the Aristotelian Boolean Logic, is no longer valid for visual perception, which requires an interpretative contextualization by the subject, who enacts space, rather than the detached Renaissance lived world and its posterior Cartesian formula of object-in-space-before-subject. This ambiguity which the dual logic can not account for, is more notable in auditory space [99]. Yet the sensory modes have a topographic map representation in the body whose topology is the Klein Bottle, which will be presented below and thus its ontology is the Klein Bottle Logic, rather than a dualistic logic.

The term topographic maps refer to the existence of maps of the sensorial stimuli on the brain. In particular, a visual map refers to the existence of a non-random relationship between the positions of neurons in the visual centers of the brain (e.g. in the visual cortex) and the values of one or more of the receptive field (RF) properties of those neurons. The term is usually qualified by reference to the property concerned: thus the term retinotopic map refers to the orderly mapping of RF position in retinotopic coordinates in a brain region; orientation map refers to the orderly mapping of orientation preference and ocular dominance map refers to the orderly variation in relative preference for stimuli delivered to the contralateral vs. the ipsilateral eye. These quantities can be assigned to neurons in primary visual cortex on the basis of appropriate tests with physiological stimuli such as moving light or dark bars, or sine wave gratings, presented to one or both eyes while the activities of cells are monitored with techniques such as extracellular single unit recording, optical imaging, two-photon confocal imaging or high resolution fMRI. These are not the only quantities that may be mapped: in principle any well-characterised receptive field property is a candidate for mapping in any of the visual areas [82].

The issue of topographic maps lead Penfield to discover by stimulation of areas of the cortex to identify the surface areas of the body associated to them. This lead to identify the existence of

contralateral (i.e. on the opposite side to another structure) maps of the cortex, one being somatosensory and the other being motoric; thus, sensation and action are integrated and seemingly lateralized in a feedback loop, yet as if functionally separate in each surface of the hemilateral hemispheres.

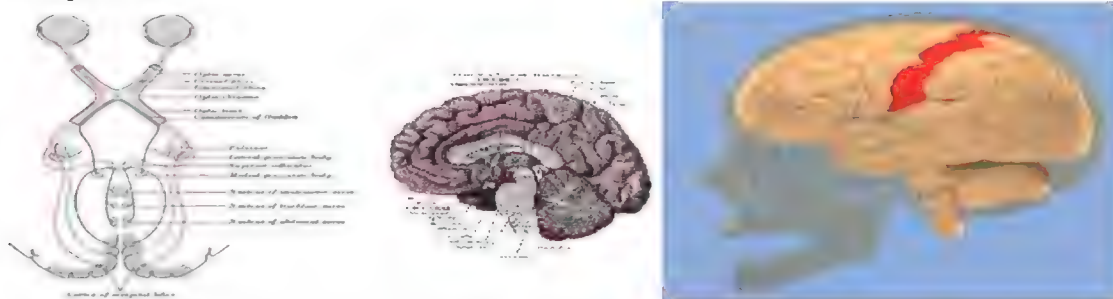


Figure 52. Left: sketch of the visual system anatomy (after Gray's Anatomy); Center and Right: the sensorymotor cortex; right figure licensed by eBodyParts3D, © The Database Center for Life Science licensed under CC Attribution-Share Alike 2.1 Japan".

Thus the cortical homunculus was discovered, a topographic 2D map associated to the primary motor cortex and the primary somatosensory cortex, i.e., the portion of the human brain directly responsible for the movement and exchange of sensory (in the left brain) and motor (in the right brain) information of the body. It can be thought as an anatomical-physiological section of the integrated sensorial modes on the sensorymotor cortex, whose topographic map has the topology of the Klein bottle [29].

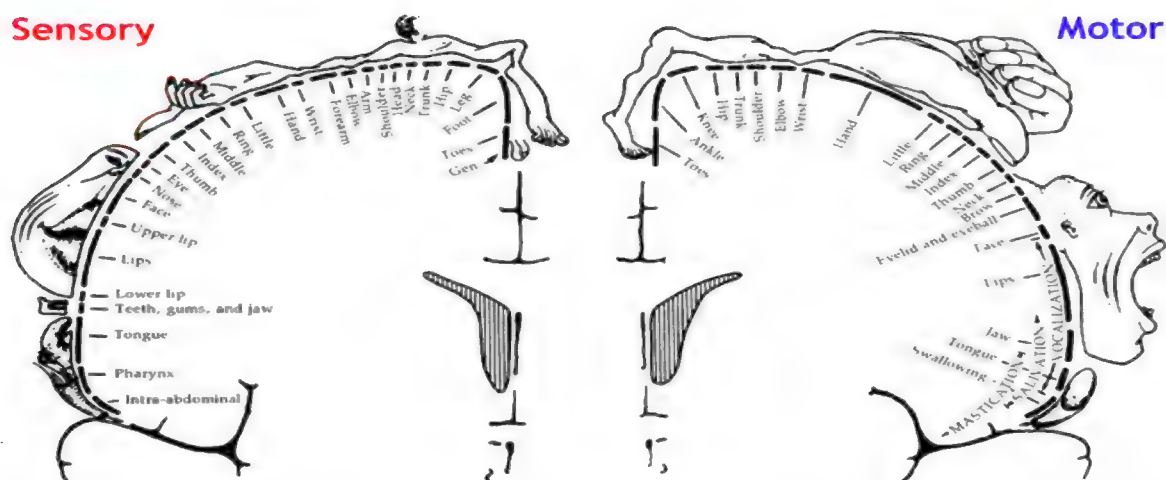


Figure 53. The homunculus; the apparent distortion of some parts of the cortical coding of the somatosensory input is the real distortion.

One striking aspect of the homunculus, is that on the left and right hemispheres, is the superposition of the motoric and sensorial locus. Alike the integrated crosslateral anatomy-physiology of vision, which is functionally representable by a Möbius-Klein bottle, with a hole playing the role of the X-cross of fig. 51:

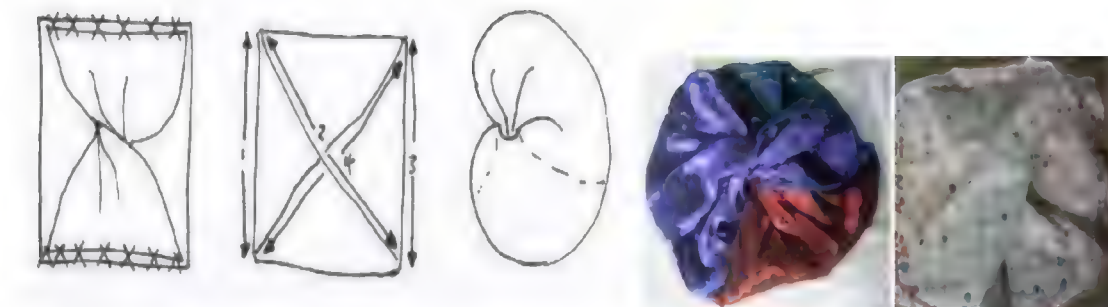


Figure 54. The Möbius Kleinbottle, a posture of the Klein Bottle, is easily constructed using three squares. This is the model described in Lewis Carol's *Mein Herr*. Two squares are joined together to make the twisted Möbius strip to finally stitch a third square as in B: Möbius Klein Bottle in C and D. The model in C made with three different coloured squares shows the twist in the purple square. In D, a model is of the same design and is offered for the reader to sew the third handkerchief onto the continuous edge of the Möbius strip. Being a fabric with a grid printed on it, the orientation of the diagonal and orthogonal weave is revealed. Reproduced with the kind permit of the author, Dr Melanie Purcell [100].

The integration of this superposed somatosensory and motoric topographic representations is produced by the corpus callosum, and that indeed we have a topological identification of the retina with the cortex, is given by the homunculus, while the cross-lateral identification is very much the design of the body. But then it natural to hypothesize that the hemilateral superposition of the motoric and somatosensory maps have precisely the non-orientable topology of the Möbius -Klein Bottle in which we can see each surface as being the local side of the globally one-sided Klein Bottle surface. Thus, the integrated feedback of the homunculus is that of the fundamental 2:1 resonance. For the physiological integration to occur, two half-octaves are playing it, each of them for the motoric and somatosensory cognitive modes, so that low and high frequencies signals are processed *as if* separate to each cognitive mode, and further integrated in the 2:1 resonance and superposed in higher octaves. Of course, as well known, both modes occur in quantized time, since elementary time frames both for perception and for action are the case. As we said before, it is the Weber-Fechner law that rules the distinctions in the scales above the elementary thresholds [101]. Furthermore, the hemilateral brain asymmetries can be, in this setting, explained by the 2:1 resonance, since the information has to cross twice the corpus callosum for establishing a cognitive frame for integrated sensation and action. Already in the established paradigm for hemilateral functional asymmetries, it is postulated that they involve small qualitative differences (as shown in the setting of the Weber-Fechner law for just noticeable differences) rather than all or nothing differences.

So the logic of the cortical integration is non-Boolean. In fact, it is postulated that the brain asymmetries arise from fundamental physical asymmetries [102], of which chirality is most basic; already, chemical chirality can be linked to the non-orientability of the Möbius strip through enantiomerism; furthermore, the current paradigm of chemistry stresses the topology of chemical configurations; we have discussed this issues above. Returning to the biological development that may establish asymmetries, it is postulated that they correspond to different rates of maturation (so biological clocks are the case) and the changing embodiment of environmental information, and thus, the left brain has an evolutionary delay in the life of an organism to be able to learn high spatial frequencies, while the right hemisphere is earlier capable of processing low spatial frequencies [102]. Therefore, it seems plausible that an interpretation of evolutionary asymmetry can be set in terms of harmonics, and in fact, it was postulated that biological evolution complies with the Klein Bottle Logic [9,11] rather than with Boolean logic assumed to be the case by embryologists. But as much as the 2:1 resonance of the non-orientability of the Möbius Band and the Klein bottle might be the case of

the evolutionary history towards establishing functional asymmetries, it can also be postulated that the quantized time frames that are related to the actual integration of both hemispheres, can also be related to the time lags that occur with respect to the functional response to stimuli, and still to sustain the anticipative response to stimuli as in the Libet's experiments, so that anticipation is undissociable from the holonomic character of the 2:1 resonance that is the case of the non-orientable Möbius and Klein Bottle surfaces. Already in aural interpretation of sequentially played half-octaves (the tritone paradox), the holonomy of the sensorial response is associated to the non-orientability of the aural map; we shall discuss this issue below. The relation of anticipation, non-orientability, hemispheric integration and Kozyrev's theory of time was discussed in [130].

Returning to the general discussion of topographic maps, the consequences of having multiple overlaid maps of different properties were realised by Hubel and Wiesel who speculated that maps ought to be superimposed in ways that would allow combinations of features such as eye and orientation preference to be equally well represented over all positions in visual space [83]. They also speculated that continuity in the layout of columns for different features would minimise connection lengths by bringing functionally related groups of cells close together in the cortex. These two ideas form the basis for a theoretical understanding of cortical maps that has been extremely fruitful [92].

In this approach, cortical maps are regarded as a dimension-reducing projection from a set of points or manifold in a multi-dimensional feature space onto the 2D surface of the cortex. It is noteworthy, that phenomenologically, these topographic maps of the sensorium produce a mapping of highly dimensional configuration (actually 6D for visual space, two axes are needed to represent receptive field position, the horizontal and vertical coordinates in visual space, two are needed for orientation (which is usually represented as a point on a circle), plus another for eye preference and another for spatial frequency) to 2D surfaces, so that their analytical structure of and the topologies associated to them are keenly related to the way that the notion of a lived world is structured and developed.

Rather than the usual approach in theoretical physics that follows Einstein's inception of time as a fourth parameter to space, in adding dimensions to account for a phenomenology to later propose another related phenomenology to explain why these dimensions can not be empirically tested or even not manifest at all, the conception is that although many degrees of freedom may be considered, the world as is experienced, notably indicates dimension two. But then an abstraction of a primeval distinction or an abstract boundary -which we identified with the torsion field- on a 2D plane, allows not only to generate the Lorentz group, but also time waves as imaginary (i.e. related to $\sqrt{-1}$) logical states that correspond to the self-penetration of the Klein Bottle which occurs in 3D but does not occur in 4D, would we immerse the Klein Bottle in \mathbb{R}^4 erasing thus its self-penetration. We recall that it was this keeping of the singularities of self-penetration what allowed us to represent the complex electromagnetic field as a minimal surface with discontinuities and thus represent the shape of Earth's crust produced by tidal waves, later identified with harmonic wavefronts above [14].

The axes of the feature space are the parameters which vary in the maps, e.g. retinal position, orientation angle, spatial frequency etc., yet, neural networks modeling with Hebbian learning have repeatedly shown that reduction to 2D is achieved, though different parameter dimensions are brought into consideration and that singularities -which correspond to the change of orientability of the topographic map- naturally appear in the 2D representation, an issue of the utmost importance, to which we shall return. The projection is assumed to be made subject to the two constraints identified by Hubel and Wiesel: completeness and continuity [82]. *Completeness* specifies that all functionally relevant points in the space should be mapped; this constraint is sometimes equivalently referred to as *coverage uniformity*. *Continuity* specifies that neighbouring points in the cortex should be mapped, as far as possible, to neighbouring points in the feature space. The issue of continuity is a most crucial issue to discuss next.

In the case of orientation maps, each neuron in the primary visual cortex responds selectively to the orientation of edges and their direction of motion of stimuli presented by luminance gratings; this response is a Receptive Field (RF) of optimal orientation. Orientation preference is mapped in a systematic fashion across the cortical surface, such that neurons in adjacent columns have similar but

slightly shifted preferred orientations; each “column” being a flat 2D slab codes for a single orientation. The mapping is continuous, and characterized by point singularities, the orientation centres at which all orientations from 0° to 180° converge in a pinwheel (i.e. vortical) singularity. These maps produce RF profiles of individual neurons and cortical arrangements of parameters relevant to the RFs which are extracted from the RFs profile obtained from the self-organization of afferent inputs. Thus, the basis for the maps lies in the self-organization of inputs. The analytical basis for these maps lies in the Gabor function $R(\mathbf{x}) = A \exp(-\mathbf{x}^2/2\sigma^2) \cos(2\pi f \mathbf{e}(\theta) \cdot \mathbf{x} - \phi)$ of the 2D position vector \mathbf{x} ; A , σ , f and \mathbf{e} represent the normalized factor of response strength, width of the RF, spatial frequency and unit vector vertical to the orientation θ , respectively. We note here the presence of cosine term, which for a 2D signal already produces the Klein Bottle as the spherical harmonics expansion, discussed already. In particular, focusing on optimal orientation ϕ and phase θ , the best fit of the Gabor function with the RFs is obtained by taking the values of the parameters with $\theta \in [0, \pi]$, $\phi \in [0, 2\pi]$, as shown in the figure 56. The white and black ovals in figure 56 show the ON and OFF, of excitatory and inhibitory response of the neuron, respectively, in the RFs derived from the Gabor function which occur along the edges of the domain. Remarkably, identical configurations of ON and OFF appear along the opposite edges of the domain. Also OFF-flank and ON-Centre RFs rotate clockwise along both horizontal edges in the domain from 0° to 180° degrees of orientation, while the direction of rotation of the RFs change oppositely in the vertical directions, establishing thus the Klein Bottle. Yet, all in and all, it is, in first glance, a Boolean response ON or OFF of the cortical cells which supplemented with the states CENTRE and PERIPHERY, all together four states, produce the four state Klein Bottle logic of the representation of the RFs. Indeed, we must remark the paradoxical logic of the behaviour of the response with relation of PERIPHERY and CENTRE already noticed by Hubel and Wiesel, in the sense that the antagonism of CENTRE and PERIPHERY is non-dual, in that the effect of a stimulus on the CENTRE was countered by stimulating a cell in the surround – “as if you were telling the cell to fire faster and slower at the same time”, i.e. a *superposition* of inhibition and activation; see [83], page 41. Already the boundaries embodied in PERIPHERY have an orientation which will lead to the Klein Bottle Logic. The topology of visual space is identical if other degrees of freedom of visual space are considered, say, spatial frequency, retinotopic position and ocular dominance are introduced [92].

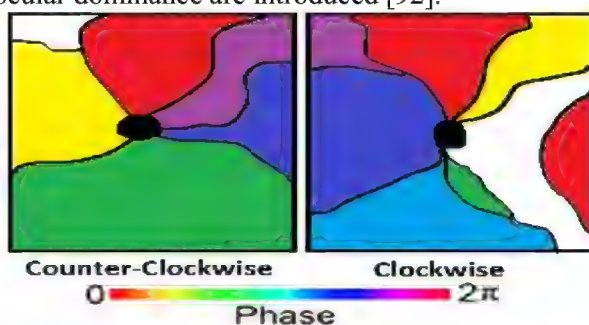



Figure 55: Representation of point singularities in the visual cortex. Each colour represents a different radial phase corresponding to an orientation column; Courtesy of Rtang 3 .

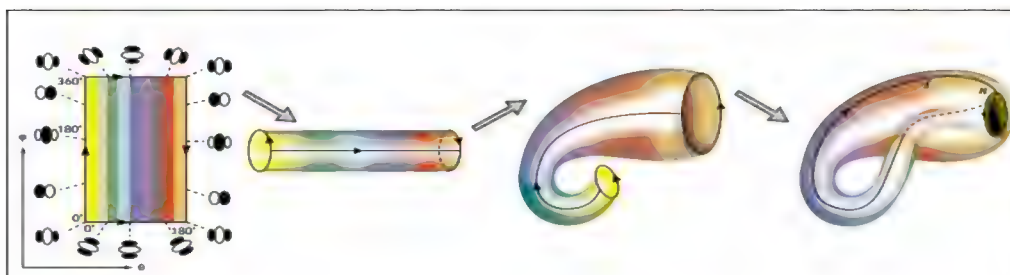


Figure 56: Demonstration of the Klein Bottle from the symmetry properties of simple cell receptive fields [30,31]. On the far left, receptive-field orientation is plotted on the horizontal axis, and receptive-field phase, which corresponds to the layout of excitatory (white) and inhibitory (black) subregions within the receptive field, is plotted on the vertical axis. Any combination of orientation and phase can be represented as a position within the rectangle. A continuous surface across which these parameters vary smoothly can (only) be constructed by joining together opposite edges of the rectangle to form a Klein bottle, as shown on the far right (reproduced, courtesy of Nick Swindale). The symmetry of the representation on the far left is identical to the representation of high-contrast pixel patches statistical analysis of natural images, which are thus found to have the Klein Bottle topology as the hidden universal form of those images [27,28].

A retinotopic map refers to the orderly mapping of receptive field position in retinotopic coordinates in a brain region. Retinotopic maps can be characterized quantitatively by the rate of change of position in the cortex as a function of position in visual space, expressed as mm/degree. This quantity is known as magnification factor (M). A high value of M means that a large area of cortex is devoted to analysing a small region of visual space. In primates, M is highest in the fovea and decreases reciprocally with visual field eccentricity. The overall relationship between visual field position and position in the cortex can be described approximately by a complex logarithmic function $\mathbf{w} = a \log(b + \mathbf{z})$ where $\mathbf{w} = (x + iy)$ gives the position in the cortex in mm, $\mathbf{z} = re^{i\theta}$, where r is the visual field eccentricity in degrees, θ is the meridional angle in radians and a and b are experimentally determined constants. This function, first proposed by Schwartz [26,32], describes the type of mapping observed in primary visual cortex (V1) of primates, including humans, including the over-representation of the fovea (the centre of gaze) and the tendency for lines of constant meridional angle and constant eccentricity to be mapped as orthogonally intersecting straight lines on the surface of the cortex. The characterization of neuronal maps via analytic (conformal) functions, has a developmental interpretation as potential flows on surfaces, subject to boundary conditions imposed by the shape of the surface. Thus, Schwartz applied the Dirichlet Principle to demonstrate that the minimization of the average magnitude of the anatomical magnification factor (by unit area) is sufficient to encode the map based on the shape of the boundary conditions. In particular, the cortical domain is clearly a rectangular strip, since the logarithmic map conformally transforms the retinal annulus to the latter cortical strip. Yet, the domain of any branch of the complex logarithm function, implicitly states that due to the domain periodicity of the imaginary axis, the orientable 2-torus, is considered to be the case, where the implicit assumption is that the complex plane is considered with the usual orientability, rather than the non-orientable topology hereby presented in our previous discussions, which is crucial to non-linear dynamics. A consequence of this is that the center of the image point of the visual map is not conserved but pushed to the boundary of the hypercolumn, so to speak, the geometrical-physiological-anatomical center is not kept as an organizing center on the anatomical-perceptual topographic map on the cortex which thus becomes asymmetric. But as proposed by Schwartz, would we choose for the topology of the domain of definition of the complex logarithm, whose principal branch has for domain the non-orientable Möbius strip, then this conservation of the centre is the case and the symmetry of the image on the cortex corresponds to the symmetry of the input on the retina [32]. We recall that the principal branch of the complex logarithm, defined on $z = re^{i\theta}$, with $\theta \in (-\pi, \pi)$, so the imaginary axis establishes a 2π -periodicity, i.e. is torus-like on gluing along the boundary of the imaginary axis domain. While the multivaluedness of the complex logarithm is dealt by defining it on a Riemann surface, which by default is orientable (either locally a 2-torus or a 2-sphere), this construction of the domain as non-orientable, places the orientable Riemann surface of visual representation as the global 2-torus on which the non-orientable Möbius strip is embedded as the 2:1 resonance that unifies both hemispheres, as is the case already of the homunculus seen as the trans-corpus callosum integration of the Möbius strip of this homunculus to give the seemingly 2-torus topology of the body. Would surgery cut across the corpus callosum which integrates both

hemispheres, the integration is no longer the case, separating the RFs into symmetric parts, as empirically verified [83].

15. The somatosensory system and the Klein Bottle

The somatosensory system embodies the cutaneous surface and pressure receptors enclosed by the body surface; thus, the perceptual modalities of it are touch and kinesthetics. The anatomical constituents are the afferent nerve fibers which originate from the peripheral receptors, ascend in the posterior funicle of the spinal cord. It forms synaptic relays in the dorsal column and the ventrobasal nuclei of the thalamus, to finally project to a topographic representation map of the body in the postneural gyrus of the cerebral cortex, known as the somatosensory area 1. In the projections of afferents from the body to the cortex, the nervous receiving system receives signals from tactile (cutaneous) and kinesthetic receptors arranged in separate cell columns oriented perpendicular to the cortical surface. For the cells that compose each modality pure cell column, it was ascertained that the peripheral RF of a neuron situated at one depth of a cortical cell column is representative of the approximate RF location of all neurons in the same cell column. Thus, the same cortical columns that are the cortical arrangement for the topographic maps of the visual mode, have a somatosensory correlate. The neurons of each cell column contribute as one unit of which the cortical maps are composed. In fact, the cell columns of the two somatosensory modalities, touch and depth) intermingle in close proximity as a mosaic made of cell columns with RFs interior to the body surface and those representing the outer skin. Thus, they represent the functional-anatomical perceptual units in which the exterior world is interiorized to the cortical system, which being of the embryonic exterior cells that by torsion folding are brought inside, is thus in terms of developmental evolution of the embryo, units that sustain a topographic map from the outer world in an inner world which is historically and determinatively also exterior. This interiorization which is physiologically reversed as a sensory system, is the embodiment of the Klein Bottle (discovered by Werner [29] and its logic, fusing the exterior and interior worlds by self-penetration. This is a mapping and realization of the ectoderm 2D surface and depth RFS which due to the columnar organization of the sensorium as anatomical-physiological units on what is thus a 2D cortex, produces the notion of a body as if 3D on what is in its historical embryological development of turning inside-out and outside-in of the embryonic egg, is a 2D phenomenology which sustains our idea of a 3D body, and still a 3D world at large.

16. The topographic visual map, the conformal inversion and non-orientability

It is quite remarkable the possibility that the identification of the complex logarithm with the visual topographic map opens to the characterization of vision also in regards to the anatomy of the eye. Indeed, the eye points inward: the photoreceptors point not to the exterior light but inwards. We recall that the retina of the human eye is the returning migration to the exterior of the body of ectodermal (i.e. exterior) tissue that by torsion folding the development of the embryo constitutes the brain. Also, the modular anatomical-physiological unit of the cortex first proposed by Mountcastle and retaken by Hubel and Wiesel, the hypercolumn of cortical neurons piled as vertical columns (i.e. normal to the orientation of the cortex, which topologically is a sphere) all responding to sensorial input as a unit, have for precursors the migration in an early developmental stage of radial glial cells forming minicolumns so turning inside out is not only the phenomenae of formation of the eye, but also of the cortex [82]. Thus, it closes a cycle in which the exterior of the developing organism has infolded to outfold for completion of the mature organism.

The fact is that the derivative of the complex logarithmic map is the function $1/z$, a particular case of a Möbius transformation in complex analysis: $z \rightarrow (az + b)/cz + d$, of one complex variable; here the coefficients a, b, c, d are complex numbers satisfying $ad - bc \neq 0$. These transformations of the extended complex plane, the Riemann sphere \mathbb{Z} , form a Lie group, isomorphic to the Lorentz group of Special Relativity. There is a one-to-one mapping between Möbius transformations and Lorentz transformation on Minkowski spacetime [66]; in fact, this is the basis for the theory of spinors as

elaborated by Penrose [103]. If we take $a=0$, $b=1$, $c=1$ and $d=0$, then we get the complex derivative of the logarithmic map. Yet, what is truly remarkable is that the Möbius transformations acting on a rectangle on the complex plane turns it inside out. In particular, the transformation $z \rightarrow 1/z$ produces the inversion and a reflection with respect to the real axis. Indeed, points outside and inside arbitrary circles on the complex plane are exchanged by the transformation $z \rightarrow 1/\bar{z}$, where $z = r \exp(i\theta)$, and the inversion is completed by a rotation transforming θ to $-\theta$, a reflection with respect to the real axis. It is easy to see that a directed segment of a line is mapped onto another directed segment with the opposite orientation [66]. If we now translate the results on C to Σ , we find that the inversion $z \rightarrow 1/z$ on C induces a rotation of Σ about the real axis through an angle of π . The inversion on C which transforms 0 to ∞ and ∞ to 0 , swaps $S=0$ with $N=\infty$ through the rotation of π about the x axis. It further transforms a neighbourhood of ∞ to a neighbourhood of 0 . Thus to study the behaviour of a map on a neighbourhood of ∞ we transform to a neighbourhood of 0 . That means that to study $f(z)$ on a neighbourhood of ∞ we need to study the behaviour of $F(z) = f(1/z)$ on a neighbourhood of 0 . This can be used to extend the concept of a branch of a complex function at 0 to a branch at ∞ . For example, $\log(z)$ has a branch at 0 , so that $F(z) = -\log(z)$ has a branch at ∞ .

Hence, the issue of divergences in real analysis, can, in principle, be surmounted by considering a complex extension in which divergences are considered analytically in the neighborhood of the origin; would this be the case, the failure of real analysis criticized in [55] would be surmounted by considering instead complex analysis, which is already the case of visual perception.

This swapping of the South Pole 0 with the North Pole ∞ , would we associate the former to the centre of the iris and the latter to the fovea (the retinal point of the projection centre of gaze), characterizes the gist of the fundamental operation which transforms the Outside world to the Interior world, the transformation of visual perception. Furthermore, vectors on the complex plane that have a certain direction, under inversion they have the opposite direction, and on a map inversion acts as depicted below.

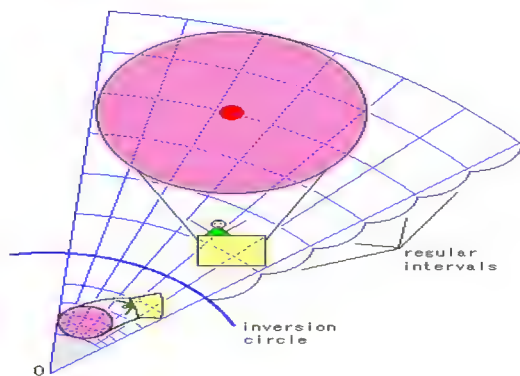


Figure 57: Action of the inversion on a map.

Thus, the inverted orientation of the photoreceptors of the retina, rather than being an anomaly, a defect of design, would there be any design in biology, a notion very strongly contested by Darwinists [139], are the very proof of the logic of the eversion by which organisms develop: they functionally operate assuring that the Outside world is mapped inside and viceversa. Their logic is the Klein Bottle. This eversion has for physiological manifestation the so called multistable perception, as is the case of the continuous transformation of the perception that arises from focusing on a 2D representation say of a cube, yielding two alternative visions, and established depth (actually the self-penetration of the Klein Bottle retinotopic map) as the primal time dimension [134,135,140] creating thus 3D space, in fact a double 3D space as we shall briefly comment further below [12,14].

Yet, as a Möbius transformation it swaps S and N , and we claimed to be the gist of the turning outside in of the world through vision. It is by introducing the Riemann sphere, the one-point compactification $\infty = -\infty = N$, allows to continuously connect N with the imaginary axis by

introducing further a π twist at ∞ . In this we see the transformation of Time, that in the complex plane rotates the real and imaginary axis, on the Riemann sphere through inversion it swaps S and N by a π rotation about the x axis but further requires a twist on N to continue to link S with $i=\sqrt{-1}$ as depicted in figure 31).

17. On the Klein bottle psychophysics of music perception

In contrast with visual space which is "out there", the omni-directional nature of the auditory field places the listener still more firmly in space: auditory space is all around – and even inside – the listener (depth is thus crucial to audition), and the body is much more a participant in space. Thus, whereas visual space can give us the illusion of detachment, the auditory mode involves distinctly the body of the listener, whose sensorial modes have the Klein Bottle for the topology of its topographic maps. In this section we shall deal with music perception, which also has a non-linear topology. This non-linear structure of aural space was empirically verified by psychologist Roger Shepard [86], which he further associated with the double helix, as in the standard model of DNA, and became a point of departure in studies in experimental psychology and music. To resume the gist of the phenomenon: 1) "Subjects interpret circularity", i.e. the re-entrance of pitch space on itself, "in the frequency doubling at the octaves" this is the fundamental 2:1 resonance, and 2) "Subjects interpret tones in an interval having a tendency or tension to move up or down on whether it is less than or greater than a half octave or tritone" ([91], page 39 and 40). This is the so-called Tritone Paradox [84,85], "which indicates that we perceive pitch intervals as ascending when in an interval between 0° and 180° , and descending when in an interval between 180° and 360° ", the 180° being the twist that produces the Möbius strip, as in the Figure 49. This is the case for the perception of two pitches of an octave, presented either sequentially or simultaneously. Topologically, this means that the interpretation of pitch as ascending or descending, for an octave, can be identified with the Möbius strip, in one local side we interpret as ascending, in the other local side, as descending. Already, musicologist Dimitri Tymoczko through mathematical arguments in terms of orbifolds, proposed that ordered tones lie on the Möbius strip [87], yet stopped short of providing a connection with the physiology of hearing. Roughly speaking, the connection is that aural perception (not necessarily restricted to music), as is the case of the sensorium's codification, has the topology of the Klein Bottle, while anatomically, the outer ear has the topology of the Möbius strip (which is built-in in the Klein Bottle) and the cochlea has the shape of a Golden Number spiral, the generic architecture for harmonics.

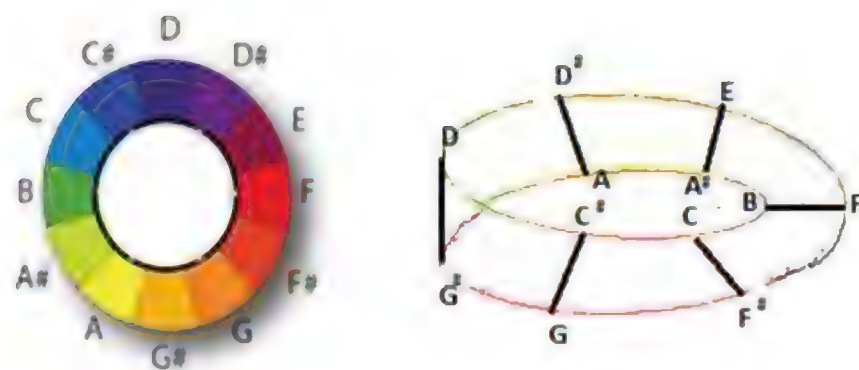


Figure 57. Left: topological representation of the chromatic aural space of an octave given by the disk-type real projective plane, represented as disk without a centre. The colours represent the synesthetic nature of perception in which the pitches are each associated with a colour according to the frequency [87]. Right: we depict without the colours the octave on the single edge of a Möbius with the opposite points joined by lines representing the tritone (half octave) perceptual identification of the lhs figure of a circular octave. This represents the fundamental 2:1 resonance: A complete rotation on the lhs circular pitch space of an octave perceptually translates into two complete rotations

in the Möbius strip, say D-D#-E-F-F#-G-G# followed by G#-A-A#-B-C-C#-D, completing the single edge of the Möbius strip. The perceptual space turns to be the Klein Bottle surface, on identifying the antipodal points –depicted by lines on joining the antipodal pitches on the edge of the Möbius strip– as perceptual identities; the Klein Bottle thus arises as the identification of the equally oriented sides of Figure 18 B, since already the identification of opposite orientations of the other two sides produce the Möbius strip. These lines represent the perceptual identification of the tritone (half-octave). Therefore, perceptual space –according to the Tritone Paradox– is a Klein Bottle surface, which has not been acknowledged before. Music perception assents to the 2:1 resonance being a lived world. We recall that Kepler, in his *Harmonices Mundi*, created an intricate model of the solar system in terms of music harmonics, further related to the Platonic solids, yet this notion of the universal and foundational character of music has been intensively patronized as deeply wrong and misleading [115].

18. Forewords

The theory we have presented has rested in topological considerations and in terms of self-referential torsion geometries; we have chosen to leave equations aside, as they appeared to be unnecessary. The universal relevance of rotational vis-à-vis linear motions has been revealed, and spinor fields rather than exclusive to microscopic physics, are universal, as claimed by Kiehn [95]. The 2:1 resonance has appeared as a fundamental organizing principle. We have shown that the evolutionary generation of stable shapes in terms of wavefronts is related to dissipation process, as a form of attenuation of compressibility by dissipating kinetic energy through heat. Harmonic fields as proposed in Fock's theory of gravitation for its foundation have shown to be indeed essential even for geophysical morphogenesis. Yet, in terms of the organization of structures in terms of the 2:1 and higher resonances, the issue of dampening is crucial. Since without these dampening processes, the persistence of the processes and structures thus generated would not be the case. Interference must be at the roots of the persistence of these processes and structures, since on the contrary, superposition of resonances would lead to their destruction.

Richard Merrick proposed in his mathematical theory of music and its cognition/ perception, that the tones in aural space are produced by a standing wave [93]. The Periodic Table of Elements of Chemistry of Mendeleev, which following its inception was claimed to be related to harmonics, has been proved to be related to Golden Mean spiral dispositions of the natural numbers (representing atomic numbers) which can be mapped, to a Klein Bottle topology for the atomic (and their stable isotopes) elements of the Periodic Table [58]. The non-orientable topology of the Periodic Table including the stable nuclides, which reflects the exclusion principle of fermions, appears to originate in the Fibonacci spiral geometry of the natural numbers that stand for the fundamental parameter of the periodic table, namely, atomic number, i.e. number of protons (electrons), and the ensuing proton number over neutron number ratio, and its relation with the Golden mean Φ ; in this theory, the Aufbau orbitals filling rules that conform to Fermi's exclusion principle are related to the periodicity properties of this spiral geometry of atomic number, and particularly to those of prime numbers [129].

Furthermore, this topology of the Periodic Table implies a unification –rather than the duality purported to be a fundamental principle for Physics– of atoms of matter and anti-matter as lying in either side of the Möbius strip or the Klein Bottle, which can be mutually inter-transformed, alike the unification of pitch as ascending or descending, in the same form. Any perturbation to an atom producing radioactive decay returns to the stable Φ (the Golden Mean) structuration on the Klein Bottle of the Periodic Table. Furthermore, the atoms and stable isotopes can be generated from a sinusoidal standing wave, as in Merrick's theory of music. Therefore, the torsioned/twisted geometry of harmonics and the Golden Mean, Φ , lie at the very basis of bodyplans (as shown by Leonardo da Vinci in his *Man of Vitruvius*), the musical, perceptual and physical/chemical world, and further to the genetic code [12], and to the Matrix Logic derived from the Klein Bottle Logic [14,15,20]. Furthermore, the natural numbers have been proved to have a rotational planar structure [112] that

allows for the potential generation of all the natural numbers and still to identify ad negatio the prime numbers. It is plausible to hypothesize that the natural numbers might be generated in toto by standing waves, at least already mathematically proved in the case of the Periodic Table and music; notably, the stratigraphic Möbius-Klein Bottle Analemma formations of geology, are produced by the harmonics of the Sun, Moon, Earth and other planets interactions [75], and structures crystalize/sediment with respect to the teleo-logical action of Φ . Indeed, it has been found that the diameters of the Analemmic counter-rotating cells on Earth's crust and in the atmosphere, have for quotient of their diameters the approximate value $\Phi \approx 1.618$ or $1/\Phi$. It is remarkable that the Cornu spiral, whose duplication by connecting both ways the dipole singularities in terms of which the Analemma is organized, were used by Huygens for evaluating the Fresnel integrals which show up in the evaluation of the diffraction intensities for the Fresnel diffraction of the light from a slit.

The essential role of standing waves is here evidenced: they provide for the generation, of structures and processes, while the Fibonacci sequence provides the meta-algorithm for assimilation of disturbances, whatever their domain may be. Indeed, already the photon which is a standing wave transaction between emitter and receptor of electromagnetic waves, manifesting as a singularity of the wave, biological, musical, geological, etc., yet in the present phenomenology, they appear as time waves from which the physical (in the ample sense) world comes to be organic (in the sense of wholeness), unseparably from the Klein Bottle, as the logophysics sustaining this organicity. In both the aural space and the Mendelev table, the Klein Bottle embodies the organicity (in the sense of holonomy) of aural and atomic/chemical material space and particularly of massless particles (photon and neutrino), while Φ appears as the asymptotics teleo-logical result, of the most fundamental meta-algorithm claimed by Johansen [113]. The Fibonacci sequence as an algorithm, embodies the physically mensurable relations, of the unfolding fusion of the world of thought with the physical world, to reach for stable manifestations; this teleo-logical process is reached by interaction with the environment, so it is an issue of contextualization, embodied by the Klein Bottle Logic. Thus, it is universal, applying to all domains, whether biological, cognitive, economical, physical, in the natural numbers, music [91], astro/geo-logical (through the Croll-Milankovitch cycles and the Möbius strips stratigraphy,, astronomical (the distribution of planets in the Solar System), and chemical/cosmological, as related to the Klein Bottle topology of Mendelev Table . In the latter case, we see that the conformation of periodicity in a graph with atomic number N as independent variable and the ratio of number of neutrons to the number of protons as dependent variable, where the periodicity of the table appears when the latter ratio converges to $1/\Phi$ following the Fibonacci sequence from which a ratio equal to 1 can be extrapolated. The latter value corresponds to the formation of more complex atoms starting from hydrogen, by the addition of α -particles, under very extreme pressures, say in the interior of a star, with a notable structure, which amounts to the turning inside-out of the electron orbital ordering, which later, in less extreme pressure conditions, turn to their usual electronic configuration proposed early in Quantum Mechanics, while the high-pressure conditions produce a neutron which itself has a Klein Bottle topology. A figurative representation of this would be that a spherical plasma from which the Solar system was thought to have been born, would have turned inside-out to constitute the planets which were further ordered into their present orbits following the Golden mean relations through the recurrence of the Fibonacci sequence, while the Sun retracted to become the center of the Solar System. Indeed, the inversion of orbitals occurring in the Periodic Table as a signature of its non orientable topology, points out to an unacknowledged eversion of the spherical $O(4)$ symmetry of the $1/r$ potential in atomic physics. We owe to an anonymous reviewer's query the motivation for the present observation.

So, on the one hand we have that the naissance of atoms from hydrogen atoms, the latter being the building blocks of the atoms in the periodic table, by inversion of their electronic orbital-like configurations on reaching for their stable organization achieved by fusion of hydrogen atoms; this is similar to the case of aural perception in which the sequence of tones perception is inverted in aural space in its Klein Bottle topology. So the dual logic is not merely the logic of the evolutionary formation of the material world, but rather a logic for generic morphology in which Inside and Outside

are fused, the Klein Bottle Logic, which we proved to be the logic of image recognition both in the body and in the recognition of natural images taken by digital devices. We have found that the turning-inside-out of an ovum, is also the logophysics by which a fertilized egg unfolds to become an embryo [11]. Furthermore, in the unfolding of the periodic structure of the material atomic/chemical world, basic also to the biological realm, the Fibonacci sequence acts as the generative meta-cognitive meta-algorithm which constrains standing waves, by dampening; as we already discussed in perturbation of atoms, this self-referential sequence produces their radioactive decay to their stable harmonic configurations. This meta-algorithm drives the Plenum to irreducible singularities; in the case of the natural numbers to evidence the prime numbers [112,113]; for a discussion on the ontology of this meta-algorithm and its role in number theory we refer to [120], who already claimed in [121] its primeval role as a universal meta-algorithm. The relation between Φ , proportions of bodyplans of animals, the genetic code and music was elaborated in (Petoukhov & He). But still more important than Φ 's guidance of body development to the proportions of lengths of bones (musical pitches, atomic weights, etc.) is the notion that it might be more fundamentally related to the gaps between them (more generically, of singularities) as the result of the contextualization with the environment; already the photon as the primeval domain of articulation of the physical and thinking realms, generates the Fibonacci sequence. In other words, Φ may be related to the domain of articulation as a teleo-logical feedback guide directing through recurrence of the Fibonacci meta-algorithm the unfoldment of the process towards stability and holonomic coherence, of music, of the material world as appears to be the case of the Periodic Table) and the freedom to move, create and recreate. (At this point we urge the reader to return to read the last two paragraphs of Section 12). This is also already the case of the Matrix Logic derived from the Klein Bottle Logic, in which the Golden Mean is associated to OR, the logical disjunction operator. This conception of the role of harmonics in the case of the gaps of bones, is related to the total null torsion of bodyplans, postulated in [64], following work in the surmountal of the Cartesian Cut and human gait [63]. Finally, inasmuch the Klein Bottle logic arises from the notion of a boundary or distinction (after Spencer Brown) which is subverted by time waves motions exchanging the dual states Inside-Inside and Outside-Outside, we have seen that whenever a fringe image is perceived, this requires that the multivaluedness of the orientation of the boundary, is dealt with the multistate Klein Bottle logic. Therefore, the issue of the discontinuity of the boundary value for the fundamental electromagnetic field propagation, has for its visual manifestation, the indefiniteness of the orientation of the phase of the signal on the boundary, producing thus the Klein Bottle, and for its geophysical manifestation, the Analemmas on the crust or still on the atmosphere, revealed in [75].

One of the foundational issues for mathematics and physics that we have dealt with in the framework of this paradigm is the non-orientable topology of the complex plane and the Riemann sphere, which has been unacknowledged before at large but by Boeyens. While physical measurements requires real Hilbert spaces and it is the spectral theory of Hilbert spaces that extends this to complex Hilbert spaces, it is the geometry of complex projective spaces (the Fubini-Study metric) of importance to quantum mechanics. There is still a torsion potential which drives two-state quantum *open* systems to manifest a single spectral value [118,119]; the setting for this is the random Brownian motions associated to the random Schrödinger equation for open systems, while the case of the linear and non-linear torsion geometry of the usual Schrödinger equation were presented in [118]; thus torsion geometries which are already intrinsic to fluid and magnetized fluids are also the case of quantum systems. Stueckelberg, on stressing the real Hilbert spaces noticed that charge required complex Hilbert spaces [122,123]. This leads to consider whether charge (following a query of an anonymous reviewer to this article) could be related to the Klein Bottle non-orientability of the complex plane.

In the last years a self-referential construction for physics and mathematics has been formulated in terms of the Dirac algebra of quantum mechanics in the framework of the Nilpotent Universal Rewrite System (NURS); it was further mapped to the double-strand model of DNA in the works by Rowlands and Hill [116,117]. While this theory was proposed to be based on fermion-boson dualism and

dualism as a universal principle, it has been argued that this alleged dualism is actually based on the Klein Bottle logophysics [12]; we already saw in this article that there is a continuity between fermions and bosons, as provided in the two first windings that generate the self-referential torsion geometry and dynamics of the mammal heart, or still the tidal morphogenesis of Earth's crustal dynamical configurations. Thus the self-referential NURS was argued to be based in the same non-orientable topology basic to our current theory. The Dirac algebra representation in NURS requires complex quaternions, and thus a double 3D space appears: The elements with real coefficients will generate a 3D position space while the imaginary ones generate charge or still angular momentum space; this stems from the fact that the only truly fundamental parameters are space, time, mass(-energy) and charge. A double 3D space is necessary for the unfolding spiral growth of molluscs without breaking [128]. For further discussions on this construction of the double 3D space, the genesis of space and its relation with the multistable visual perception which is the functionality of the Klein Bottle Logic of visual perception discussed in this article, and the standard double helix model of DNA superseded by the Klein Bottle representation fractal structure of genomes, we refer to [12]. Remarkably, the 2:1 resonance characteristic of the non-orientability of the Möbius strip and the Klein Bottle, which we have evidenced in very diverse fields and demonstrated to be a precursor to Newton's second law, appears to be the case of the human and chimpanzee already sequenced genomes, an issue that we shall discuss elsewhere and further in relation with the quantum coherence that is established in genomes due to this resonance.

Acknowledgments

It is a pleasure to express my gratitude to Prof Martin Land and Prof Larry Horwitz for their kind support, to Craig Nevin and to Mikhail Bazanov for discussions on biomechanics (with the latter in 2009). To Sonia, without her love and support this work would not have come to be but a figment of my imagination.

References

- [1] Schrödinger E 1991 *What is Life?* (Cambridge University Press Cambridge, UK)
- [2] Gordon R 2002 *The Hierarchical Genome and Differentiation Waves: Novel Unification of Development, Genetics and Evolution* (World Scientific, Singapore)
- [3] Gordon NK and Gordon R 2013 *Embryogenesis Explained* (World Scientific, Singapore)
- [4] Forgacs G and Newman S 2000 *Biological Physics of the Developing Embryo* (Cambridge Univ. Press, Cambridge, UK)
- [5] Maresin V M and Presnov E 1985 *J. Theor. Biol.* 114: 387–398
- [6] Jockusch H and Dress A 2003, From Sphere to Torus: A Topological View of the Metazoan Body Plan. *Bull. Math. Biol.* 65: 57–65
- [7] Nishii I and Miller SM 2010, Volvox: simple steps to developmental complexity? *Curr Opin Plant Biol.* Dec;13(6):646-53
- [8] Levy S 1995, *A brief history of sphere eversions. In: Making Waves* (A K Peters Ltd, MA).
- [9] Rapoport D L 2013, HyperKlein Bottle Ontopoiesis of Life and the Cosmos, 130 pages, to appear, *Analecta Husserliana* Volume CXVII..
- [10] Rapoport DL 2013. Klein Bottle Logophysics, Cell Biology, Chemistry and Body Integration, In: *Proceedings of CASYS'11, Conference on Computing Anticipatory Systems* (LIEGE, August 2011).
- [11] Rapoport D L 2011 On the Fusion of Physics and Klein Bottle Logic in Embryogenesis, Cell Biology and Evolution, *NeuroQuantology* 9 (4): 842-861
- [12] Rapoport D L 2011 Surmounting the Cartesian Cut: Klein Bottle Logophysics, The Dirac Algebra & the Genetic Code *Neuroquantology* 9 (4): 862-881
- [13] Boeyens J C A 2005 *New Theories for Chemistry* (Elsevier, Holland, 2005).

- [14] Rapoport D L 2011 Surmounting the Cartesian with Philosophy, Physics, Logic, Cybernetics and Geometry: Self-reference, torsion, the Klein bottle, the time operator, multivalued logics and quantum mechanics, *Found. Phys.* 41(1): 33-76.
- [15] Rapoport D L 2011, Torsion Fields, the Extended Photon, Quantum Jumps, The Klein Bottle, Multivalued Logic, the Time Operator, Chronomes, Perception, Semiosis, Neurology and Cognition. In: *Focus in Quantum Mechanics*, D. Hathaway and E. Randolph (eds), (Nova Science, NY).
- [16] Stern A 2001 *Quantum Theoretic Machines* (Elsevier, Amsterdam).
- [17] Russell B and Whitehead A 2011 *Principia Mathematica* (Rough Draft Printing)
- [18] Spencer-Brown G 2010 *Laws of Form* (Bohmeier Verlag, Leipzig).
- [19] Rapoport D L 2010 Torsion, propagating singularities, nilpotence, quantum jumps and the eikonal equations. In: *Computing Anticipatory Systems, Proceedings CASYS'09*, Daniel M. Dubois (ed.), AIP Conf. Series 1303. (Springer, Berlin).
- [20] Rapoport DL 2009, Torsion Fields, the Extended Photon, Quantum Jumps, the Eikonal Equations, the Twistor Geometry of Cognitive Space and the Laws of Thought. In: *Ether, Spacetime and Cosmology vol. 3: Physical Vacuum, Relativity and Quantum Mechanics*. Duffy M and Levy J (editors): 389-457, (Apeiron Press, Quebec, Canada).
- [21] Pettigrew J B 1908 *Design in Nature* (Longmans, London).
- [22] Pettigrew, J B 1874 *The physiology of circulation in plants, in the lower animals, in man* (MacMillan, London).
- [23] Lehar S 2003 *The World in our Heads: A Gestalt View of the Conscious Experience*, (Lawrence Erlbaum Associates, London)..
- [24] Von Helmholtz H 1971, *Selected Writings of Hermann von Helmholtz* Kahl, Russell, ed. (Wesleyan Uni. Press, USA).
- [25] Hartmann W 1997 *Signals, Sound and Sensation* (Springer, Berlin).
- [26] Schwartz E L 1980 Computational Anatomy and Functional Architecture of Striate Cortex: A spatial Mapping Approach to Perceptual Coding, *Vision Research* 20. pp. 6.85.
- [27] Carlsson G, Tigran I, de Silva V and Zomorodian A 2008, On the Local Behavior of Spaces of Natural Images *Int J Comput Vis* 76: 1–12
- [28] Carlsson G 2009, Topology and data, *Bulletin AMS Math. Soc.* 46, No 2: 255 308.
- [29] Werner G 1968, The Topology of the Body Representation in the Somatic Afferent Pathway. In: *The Neurosciences Second Study Program*, 605-617, Schmitt FO (editor), Rockefeller University.
- [30] Swindale N 1996, Visual Cortex: Looking into a Klein Bottle, *Current Biology* 6 No 7: 776–779
- [31] Tanaka S 1997, Topology of Cortex Visual Maps, *Forma* 12: 101-108
- [32] Schwartz E L 1977 Afferent maps in the primate visual cortex and the generation of neuronal trigger features, *Biological Cybernetics* 28: 1-14
- [33] Scott A 2002 *Encyclopaedia of NonLinear Science* (Routledge, Taylor and Francis, London)
- [34] Rayleigh J 1984 *Theory of Sound* (MacMillan, London)
- [35] Jenny H 2001 *Cymatics: The Study of Wave Phenomena*, (B& W Publishers, London)
- [36] Arévalo J, Aguilar L M, Robledo-Sánchez C, Arroyo Carrasco M L, Mendez Otero M, The principle of superposition for waves, (preprint, arXiv:1005.3237)
- [37] Hoffman R 1995 *The Same and Not the Same* (Columbia University Press, New York)
- [38] Sokolov I 1973 Topological Methods in Stereochemistry. *Russ Chem Rev* 42 (6).
- [39] Mae-Won Ho 1994, What is (Schrödinger's) Negentropy? *Modern Trends in BioThermoKinetics* 1994,
- [40] McClare C W F 1971 Chemical machines, Maxwell's demon and living organisms, *J Theor Biol* 30: 1-34
- [41] Stapien M et al 2007 *Angewandte Chemie* 119, Issue 41, pgs. 7859-8047
- [42] R. Van Wijk (2001), *Journal of Scientific Exploration*, 15, No. 2, pp. 183–197.
- [43] <http://www.cs.sandia.gov/newsnotes/2009newsnotes.html#klein>

- [44] Herges R 2006, *Chem. Rev.* 106, 4820-4842
- [45] Rzepa R (2005) *Chem. Rev.* 105, 3697-3715
- [46] Ingber D. The architecture of life. *Scientific American*, January, 1998: 44-57
- [47] Oschman J L 2000, *Energy Medicine: The Scientific Basis* (Harcourt Brace, Edinburgh)
- [48] Gurwitsch A.G. 1932 and 1959 *Die mitogenetische Strahlung* (Fischer Verlag, Jena)
- [49] Gariaev P, Gariaev P.P., Birshtein B.I., Iarochenko A.M, Marcer P.J., Tertishny G G, Leonova E A, Kaempfer U 2001, The DNA-wave biocomputer. «CASYS» – International Journal of Computing Anticipatory Systems (ed. D.M.Dubois), Liege, Belgium, 10, 290-310
- [50] Albrecht-Buehler, G 1992 *Proc Nat. Academy of Sciences USA* 89, 8288–8292
- [51] Svensson B 2008 A Multidimensional Filtering Framework with Applications to Local Structure and Image Enhancement, (Linköping Studies in Science and Technology no. 1171, Sweden)
- [52] House M R 1995, Orbital forcing time scales: an introduction, *Geological Society, London, Special Publications* 1995; v. 85; p. 1-18
- [53] Freedman W and Schreiner M 2009 *Spherical Functions of Mathematical Geosciences: A Scalar, Vectorial, and Tensorial Setup*, (Springer, Berlin)
- [54] Wahr J 1996, *Geodesy and Gravity: Class Notes*, (Samizdat Press, Denver)
- [55] Yi Lin Y and Yong Wu 2002, *Beyond Nonstructural Quantitative Analysis - Blown-Ups, Spinning Currents and Modern Science*, (World Scientific, Singapore)
- [56] Bennett K D 1990 Milankovitch cycles and their effects on species in ecological and evolutionary time, *Paleobiology*, 16 (1), 1-21
- [57] Clark II G 1974 Growth Lines in Invertebrate Skeletons *Annual Review of Earth and Planetary Sciences* 2: 77-99
- [58] Boeyens J C 2010, *Chemical Cosmology* (Springer, Berlin).
- [59] Veblen O & Hoffmann H 1930, Projective Relativity, *Phys.Rev.* (36) 810–822.
- [60] Fischer P, Iofe A, Jacobson D L, Arif F, Meze F 2000 4π -Periodicity of the spinorwave function under space rotation, *Nuclear Instruments and Methods in Physics Research A* 440; 575--578.
- [61] Rapoport D L and Sternberg S 1984 On the interactions of spin with torsion *Annals of Physics* vol. 158, 447-475 .
- [62] Rapoport DL 2005 On the unification of geometric and random structures through torsion fields: Brownian motions, viscous and magnetic fluid-dynamic, *Found. Phys.* 35, no.7, 1205-1244
- [63] Nevin C 2010 Initiation and Control of Gait Principles: A Mathematically Animated Model of the Foot, Dissertation.com, Boca Raton, 2010.
- [64] Nevin C and Rapoport D L. The self-referential bodyplan; Klein Bottle bauplans, to appear.
- [65] Ling G. A 1982 *Revolution in the Physiology of the Living Cell*. (Krieger Pubs, Malabar, FL)
- [66] Needham T 2009, *Visual Complex Analysis* (Clarendon Press).
- [67] Kozyrev N A 1991 *Selected Works*, published by Leningrad State Univ., 1991. 488 p.
- [68] Kozyrev N A and Nasonov VV 1980,. On some properties of time discovered by astronomical observations. In: *Proyavleniye Kosmicheskikh Faktorov na Zemle i v Zvezdakh* (Manifestation of Cosmic Factors on the Earth and in Stars). Problemy Issledovaniya Vselennoy (The Problems of Studying the Universe). 9th issue, pp.76-84. Moscow-Leningrad (in Russian)
- [69] Hayasaka H and Takeuchi S 1989 Anomalous weight reduction on a gyroscope's right rotations around the vertical axis on the earth. *Physical Review Letters* 63, N.25, pp.2701-270
- [70] Levich A P 1996 *A Substantial Interpretation of N.A. Kozyrev's Conception of Time*. On the Way to Understanding the Time Phenomenon: the Constructions of Time in Natural Science. Part 2. The "Active" Properties of Time According to N. A. Kozyrev . Singapore, New Jersey , London, Hong Kong: World Scientific. 1996. Pp. 109-134 (World Scientific, Singapore) p. 1-42

- [71] Torrent-Guaspar F, Kocica M, Corno M, Komeda M, Carrera-Costa M, Flotats F, Cosin-Aguillar A, Wen Han J 2005, Towards new understanding of the human heart, *Eur. J. Cardiothorac. Surg.* 27, 191-201, and references therein;
- [72] Nasiraei-Moghaddam, A., Gharib, M., "Evidence for the existence of a functional helical myocardial band", *Am J Physiol Heart Circ Physiol* 296: H127H131, 2009;
- [73] Grosberg A 2008 A Bio-Inspired Computational Model of Cardiac Mechanics: Pathology and Development, Ph.D. thesis, Caltech
- [74] Rapoport D L 2007 Torsion Fields, Cartan-Weyl Space-Time and State-Space Quantum Geometries, their Brownian Motions, and the Time Variables, *Foundations of Physics* 37, nos. 4-5, 813-854, 2007
- [75] Overton H, www.PorOgle.blogspot.com
- [76] Ferronsky V I and Ferronsky S V 2010 *Dynamics of the Earth Theory of the Planet's Motion Based on Dynamic Equilibrium* (Springer, Berlin)
- [77] Milankovic M 1941, *Canon of insolation and the ice-age problem: (Kanon der Erdbestrahlung und seine Anwendung auf das Eiszeitenproblem)* Belgrade (Israel Program for Scientific Translations, Jerusalem, 1969)
- [78] Schwarzscher W 1993 *Cyclostratigraphy and the Milankovitch Theory* (Elsevier, Amsterdam)
- [79] Marmanis H 2000, Analogy between the electromagnetic and hydrodynamic equations: Application to turbulence, Ph.D. thesis, Brown University
- [80] Guyone E, Hulin P and Petit L 2001, *Hydrodynamique physique*. Deuxième édition, (InterEditions / CNRS Éditions, Paris)
- [81] de Boer P and Trabucchi J 2012, Orbitally forced sedimentary rhythms in the stratigraphic records: is there place for tidal forcing? *Sedimentology* 59 (2):379-392
- [82] Smith C M U, B 2008, *Biology of Sensory Systems* (Wiley, Blackwell, UK)
- [83] Hubel D 1988, *Eye, Brain and Vision*, (Scientific American Library, NY)
- [84] Deutsch D, 1987, The tritone paradox, *Percept and Psychology* 42 :563-567
- [85] Deutsch D 1988, The semitone paradox, *Music Perception* 6: 115-134
- [86] Shepard RN 1964. Circularity in judgments of relative pitch. *Journal of the Acoustical Society of America*, 36, 2346-2353
- [87] Tymoczko D 2006, The geometry of musical space, *Science* 7 July 313 (5783): 72-74
- [88] Marinelli R, Branko F, van der Zee H, McGinn A, Marinelli A 1995, The heart is not a pump: A refutation of the pressure propulsion premise of heart function, *Frontier Perspectives* (Journal of the Center for Frontier Sciences at Temple University in Philadelphia, Pa.)
- [89] Bóna A and Slawinski MA 2011, *Wavefronts and rays as characteristics and asymptotics*. (World Scientific, Singapore)
- [90] Fock V 1964, *The Theory of Space, time and Gravitation* (Macmillan, London)
- [91] Merrick R 2009, *Interference Theory: A grand scientific Musical Theory* (USA)
- [92] Durbin R and Mitchson G 1990, A reductional dimension framework for understanding cortical maps, *Nature* 343, July 15, 644-547
- [93] Bateman H 1995, *Electrical and Optical Wave Motion*, (Dover, N.Y.)
- [94] Cartan E 1966, *The Theory of Spinors* (Dover, New York, p. 39)
- [95] Kiehn R 2009 *Non-Equilibrium Systems and Irreversible Processes; Adventures in Applied Topology* Vol. 5 Topological Torsion and Macroscopic Spinors from a Perspective of Continuous Topological Evolution (Lulu, USA)
- [96] Bohm D 2002 Wholeness and the Implicate Order (Routledge and Kegan, London).
- [97] Heelan P 1989 *Space Perception and the Philosophy of Science* (University of California Press, Berkeley).
- [98] Luneburg R 1948 *Mathematical Analysis of Binocular Vision*, (Edward Brothers, USA)
- [99] Varèse E 1959 Spatial Music, in Schwartz & Childs (eds.): *Contemporary Composers on Contemporary Music*, p205.
- [100] Purcell M C 2006 *Towards a new era: epistemological resolution analysis by through and from*

- the Klein bottle wholeness*, Ph.D. Thesis, Department of Philosophy, University of Newcastle, Australia.
- [101] Stevens S S 1975 *Psychophysics: Introduction to its Perceptual, Neural and Social Prospects*, ed. G. Stevens (Wiley, New York)
 - [102] Hellige J B 1993 *Hemispheric Asymmetry: What's Right and What's Left* (Harvard University Press, Cambridge, USA)
 - [103] Penrose R and Rindler W 1996 *Spinors and Spacetime vol. I* (Cambridge University Press, UK)
 - [104] Prigogine I 1991 *From Being to Becoming* (Freeman, San Francisco)
 - [105] Lin Y 2008 *Systemic Yoyos: Some Impacts of the Second Dimension* (CRC Press, Boca Raton, FL, USA)
 - [106] Lin Y and OuYang S 2010 *Irregularities and Prediction of Major Disasters* (CRC Press, Boca Raton, FL USA)
 - [107] Burchfield J 1976 *Lord Kelvin and the Age of Earth* (University of Chicago Press, Chicago)
 - [108] Kiehn R 012/29/89 to 04/12/95 Notes on Minimal Surfaces and Surfaces with Pinch Points: Applications to Electromagnetism Following Bateman.
 - [109] Penrose R 2002 *Shadows of the Mind* (Oxford University Press, UK)
 - [110] Gutzwiller M 1990 *Chaos in Classical and Quantum Mechanics* (Springer, Berlin)
 - [111] González-Díaz P and Alonso-Serrano A Observing other universe through ringholes and Klein Bottle Holes, In: Towards New Paradigms, Proc of the Spanish Relativity Meetingm (29 August–2 September) AIP Conf. Proc. 1458, pp. 311-314 (Springer, Berlin)
 - [112] Johansen S E 2010 Complete Exposition of Non-Primes Generated from a Geometric Revolving Approach by 8x8 Sets of Related Series, and thereby *ad negativo* Exposition of a Systematic Pattern for the Totality of Prime Numbers'', *Journal of Dynamical Systems and Geometric Theories* 8 (2): 101-171
 - [113] Stein Johansen 2013 *Outline of a Differential Epistemology*,. English translation of the Norwegian, published by University of Trondheim (1991). To appear
 - [114] Petoukhov S and He M 2011 *Symmetrical Analysis Techniques for Genetic Systems and Bioinformatics: Advanced Patterns and Applications* (IGI Global Pubs, USA).
 - [115] Stephenson B 1994 *Kepler's Harmonics: the Music of the Spheres* (Princeton Univ. Press, Princeton, USA)
 - [116] Rowlands P 2009 *From Zero to Infinity: the Foundations of Physics* (World Scientific, Singapore)
 - [117] Hill V and Rowlands P 2008 Nature's Code, in Computing Anticipatory Systems. In: CASYS'07—Eighth International Conference; Dubois D M (editor) American Institute of Physics Conference Proceedings. Vol. 1051, pp. 117-126 (Springer, Berlin)
 - [118] Rapoport D L 2007 Torsion Fields, Cartan-Weyl Space-Time and State-Space Quantum Geometries, their Brownian Motions, and the Time Variables, *Foundations of Physics* vol. 37, nos. 4-5, 813-854
 - [119] Rapoport D L 2007 On the state-space and spacetime geometries of geometric quantum mechanics, In: *Foundations of Probability and Physics IV*, Proceedings of the Conference, Center for Interdisciplinary Mathematics and its applications, Univ. of Vaxho, Sweden, Khrennikov A and Adenier G, American Institute of Physics Conference Series (Springer, Berlin)
 - [120] Johansen S 2010 Fibonacci Generation of Natural Numbers and Prime Numbers In: Proceedings of the Third International Conference on Lie-admissible Treatment of Irreversible Processes (ICLATIP - 3) Santilli R M (editor), Kathmandu University, Nepal, April (2011)
 - [121] Johansen S 2006 Initiation of 'Hadronic Philosophy', the Philosophy Underlying Hadronic Mechanics and Chemistry. *Hadronic Journal* 29: 111-135
 - [122] Stueckelberg E C G 1960 Quantum theory in real Hilbert space, *Helv. Phys. Acta* 33, 727
 - [123] Adler S L 1995 *Quaternionic quantum mechanics and quantum fields* (Oxford, New York)

- [124] Kauffman L 2012 *Laws of Form - An Exploration in Mathematics and Foundations*, In: <http://homepages.math.uic.edu/~kauffman/Laws.pdf>
- [125] Varela F 1979 *Principles of Biological Autonomy* (North Holland, Amsterdam)
- [126] Kauffman L 1978 De Morgan Algebras, Completeness and Recursion, In: Proceedings of the Eighth International Symposium in Multiple Valued Logics 82-86
- [127] Rosch E, Varela F and Thompson E 1991 *The Embodied Mind* (MIT Press, Cambridge)
- [128] Illert C 1992 *Foundations of Theoretical Conchology* (Hadronic Press, Palm Harbor, FL)
- [129] Boeyens J C A and Levendi D 2008 *Number Theory and the Periodicity of Matter* (Springer, Berlin)
- [130] Rapoport D L 2011 Self-reference, the Möbius and Klein bottle surfaces, multivalued logic and cognition, *International Journal of Computing Anticipatory Systems* 23, 103-113
- [131] Husserl E 1970 *Logical Investigations Part II* (Routledge and Kegan, London)
- [132] Ortiz C H 2003 Incomplete Symbols, Dependent Meanings, and Paradox, In: *Husserl's Logical Investigations*, Daniel O. Dahlstrom (ed.) pp. 69-93 (Dordrecht: Kluwer)
- [133] Kauffman L 2013 *Knots and Physics* (World Scientific, Singapore)
- [134] Rosen S 2004 *Dimensions of Apeiron: A Topological Phenomenology of Space, Time and Individuation*, (Rodopi, New York)
- [135] Rosen S 2006 *Topologies of the Flesh: A Multidimensional Exploration of the Lifeworld* (Ohio University Press, Ohio)
- [136] Kiehn R 2003 Propagating Topological Singularities in the lightcone: the Photon <http://www.findthatpdf.com/search-3518144-hPDF/download-documents-spie2ext.pdf.htm>
- [137] Kiehn R 2011 *Topological Torsion and Macroscopic Spinors Vol. 5 Non-equilibrium Systems and Irreversible Processes* (Lulu, USA)
- [138] Post J 1995 *Quantum Reprogramming: ensembles and single systems, a two-tier approach to quantum mechanics* (Kluwer, Amsterdam)
- [139] Dawkins R 1996 *The Blind Watchmaker: Why the Evidence of Evolution Reveals a Universe without Design* (W W Norton, UK)
- [140] Merleau Ponty M 1962 *Phenomenology of Perception* (Routledge & Kegan Paul, London)
- [141] Durbin R and Mitchinson (1990) *A dimension reduction framework for understanding cortical maps* *Nature* **343**, 644 - 647 (15 February 1990)